Optics for AO

Lecture 2

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Levels of models in optics

Geometric optics - rays, reflection, refraction

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Physical optics (Fourier optics) - diffraction, scalar waves

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Electromagnetics - vector waves, polarization

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Quantum optics - photons, interaction with matter, lasers

Goals of this lecture

- Review of Geometrical Optics
  - Understand the tools used for optical design of AO systems
  - Understand what wavefront aberrations look like, and how to describe them
  - Finish with a characterization of the aberrations caused by turbulence in the Earth's atmosphere

An "aside" on teaching methods

- The traditional lecture is far from the ideal teaching tool
  - Researchers on education study these things rigorously!
- I can't "pour knowledge into you"
- It is you who must actively engage in the subject and assimilate it in a manner meaningful to you
- We will emphasize active learning and understanding the unifying concepts of adaptive optics

Equations vs. Concepts

- Difference between manipulating equations and being able to analyze unfamiliar situations
- Exams will include conceptual problems as well as traditional computational problems
- Example: what are the physical reasons why astronomical adaptive optics is easier at longer observing wavelengths?

Some tools for active learning

- In-class conceptual questions will aim to engage you in more active learning and provide me with feedback on whether concepts are clear
  - I will pose a short conceptual question (no calculations)
  - I will ask you to first formulate your own answer, then discuss your answer with two other students, finally to report your consensus answer to the class
- Concept maps
- Projects (~2 people to a project, formulate your own questions and answer them) later in the quarter
- Some web-sites & books about teaching and learning:
  - http://ic.ucsc.edu/CTE/teaching/
  - http://teaching.berkeley.edu/compendium/
  - How People Learn, Bransford, Brown, and Cocking, Editors; National Research Council, National Academy Press

Textbooks

- Field Guide to Adaptive Optics by Tyson and Frazier.
  – Bay Tree Bookstore (UCSC)
  – SPIE Press http://www.spie.org
  – Amazon http://www.amazon.com
  – Please order this yourselves
- Reader available from SlugBooks at UCSC
  – http://www.slugbooks.com/ Phone 831-469-7584
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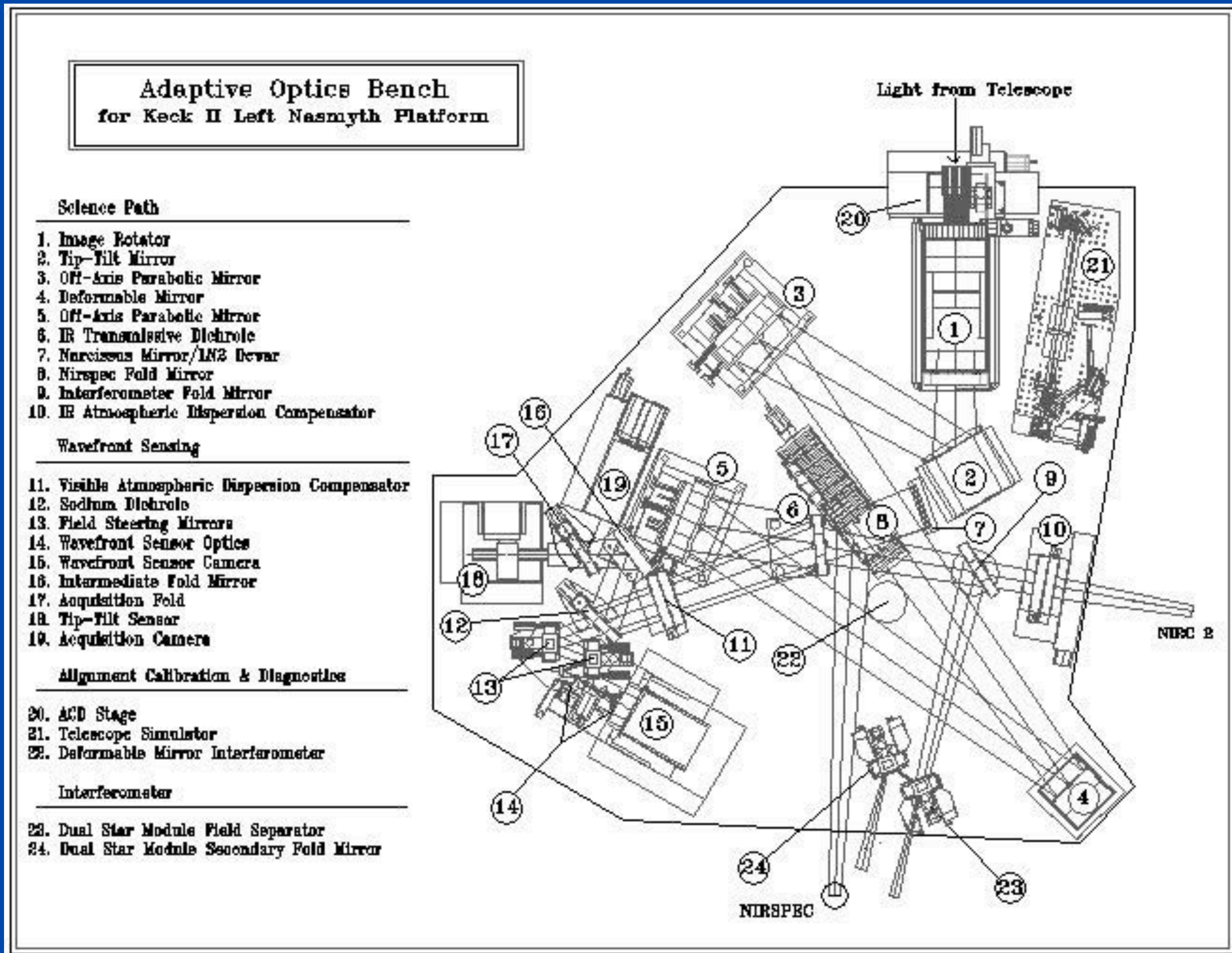
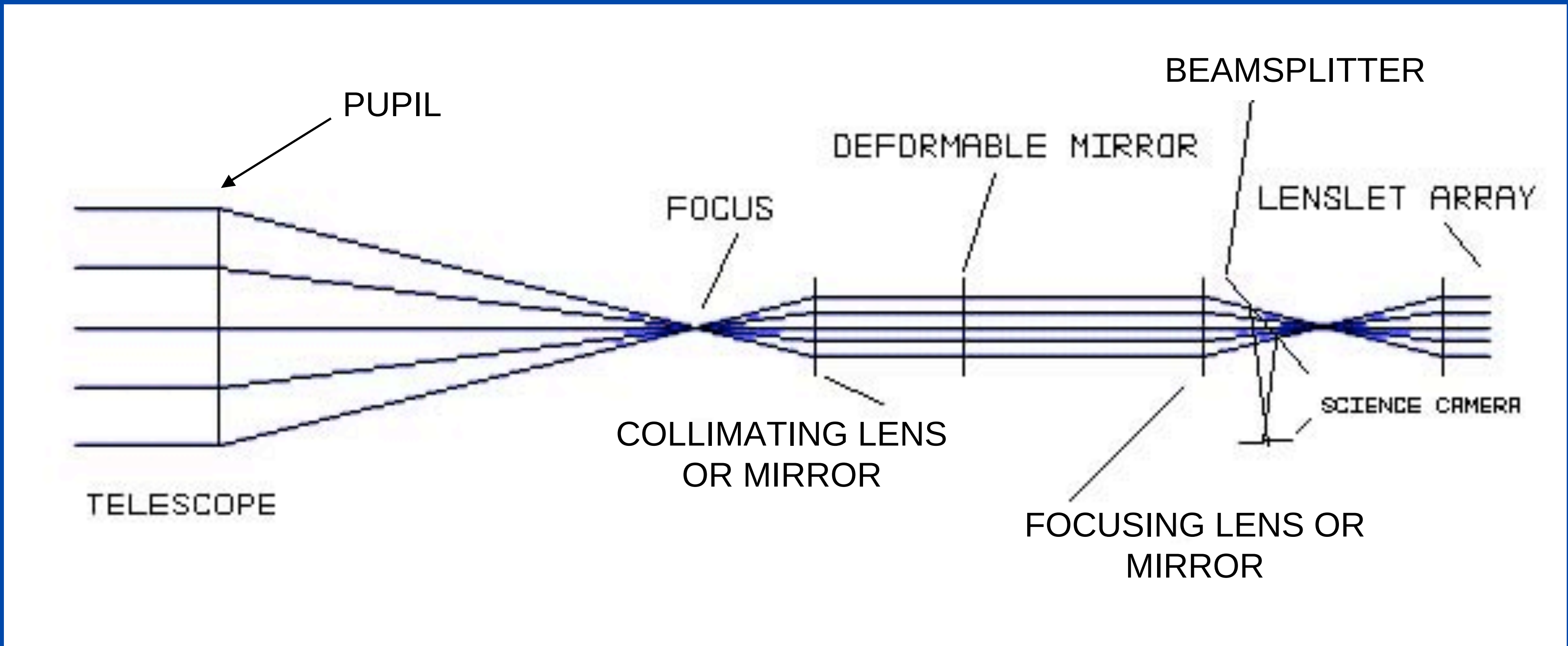
Keck AO system optical layout: why does it look like this???



Simplest schematic of an AO system



Optical elements are portrayed as transmitting, for simplicity: they may be lenses or mirrors

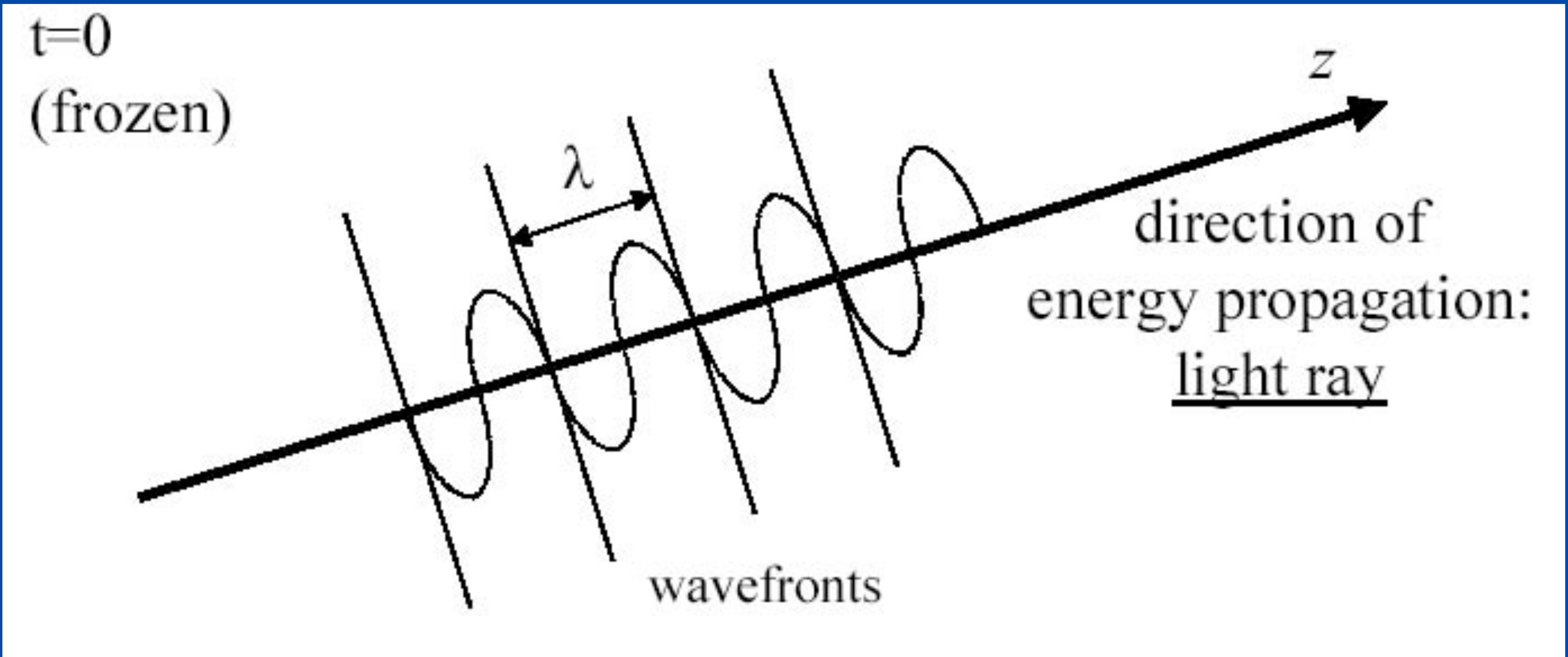
What optics concepts are needed for AO?

- Design of AO system itself:
  - What determines the size and position of the deformable mirror? Of the wavefront sensor?
  - What does it mean to say that "the deformable mirror is conjugate to the telescope pupil"?
  - How do you fit an AO system onto a modest-sized optical bench, if it's supposed to correct an 8-10m primary mirror?
- What are optical aberrations? How are aberrations induced by atmosphere related to those seen in lab?

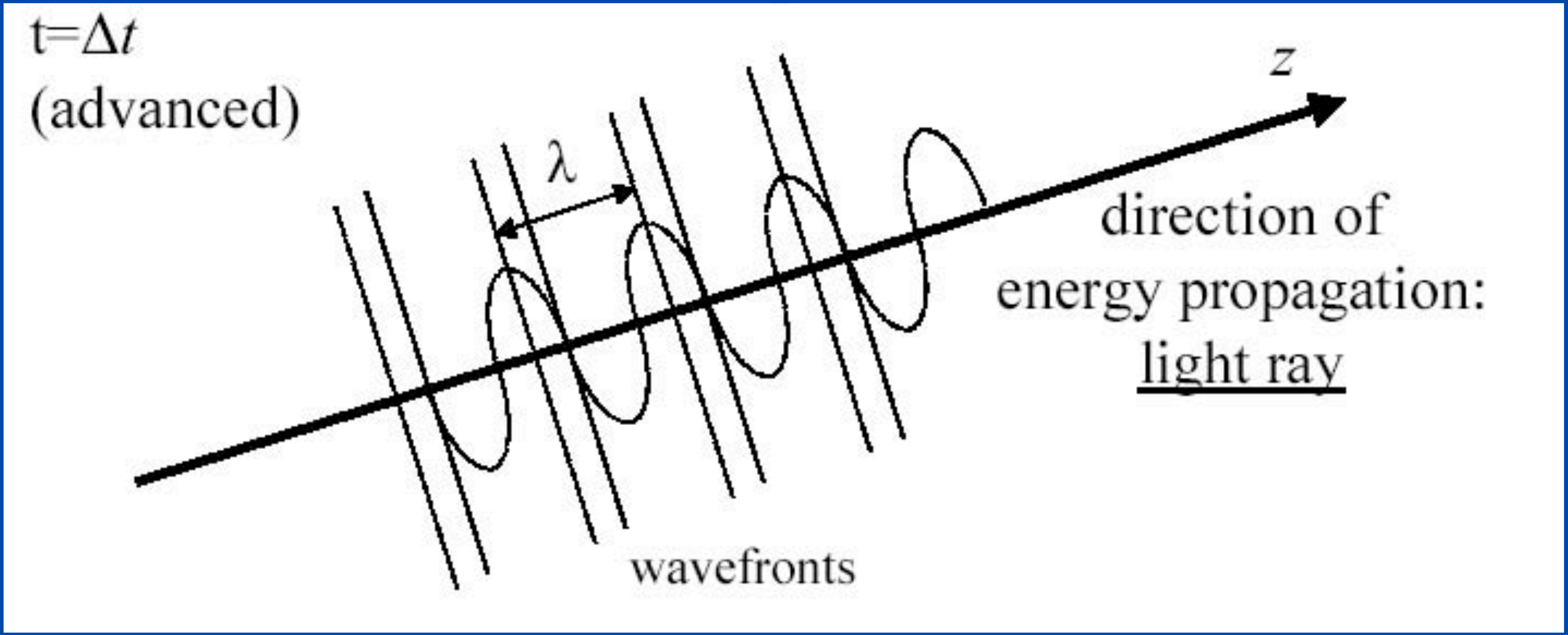
Review of geometrical optics: lenses, mirrors, and imaging

- Rays and wavefronts
- Laws of refraction and reflection
- Imaging
  - Pinhole camera
  - Lenses
  - Mirrors
- Resolution and depth of field

Rays and wavefronts

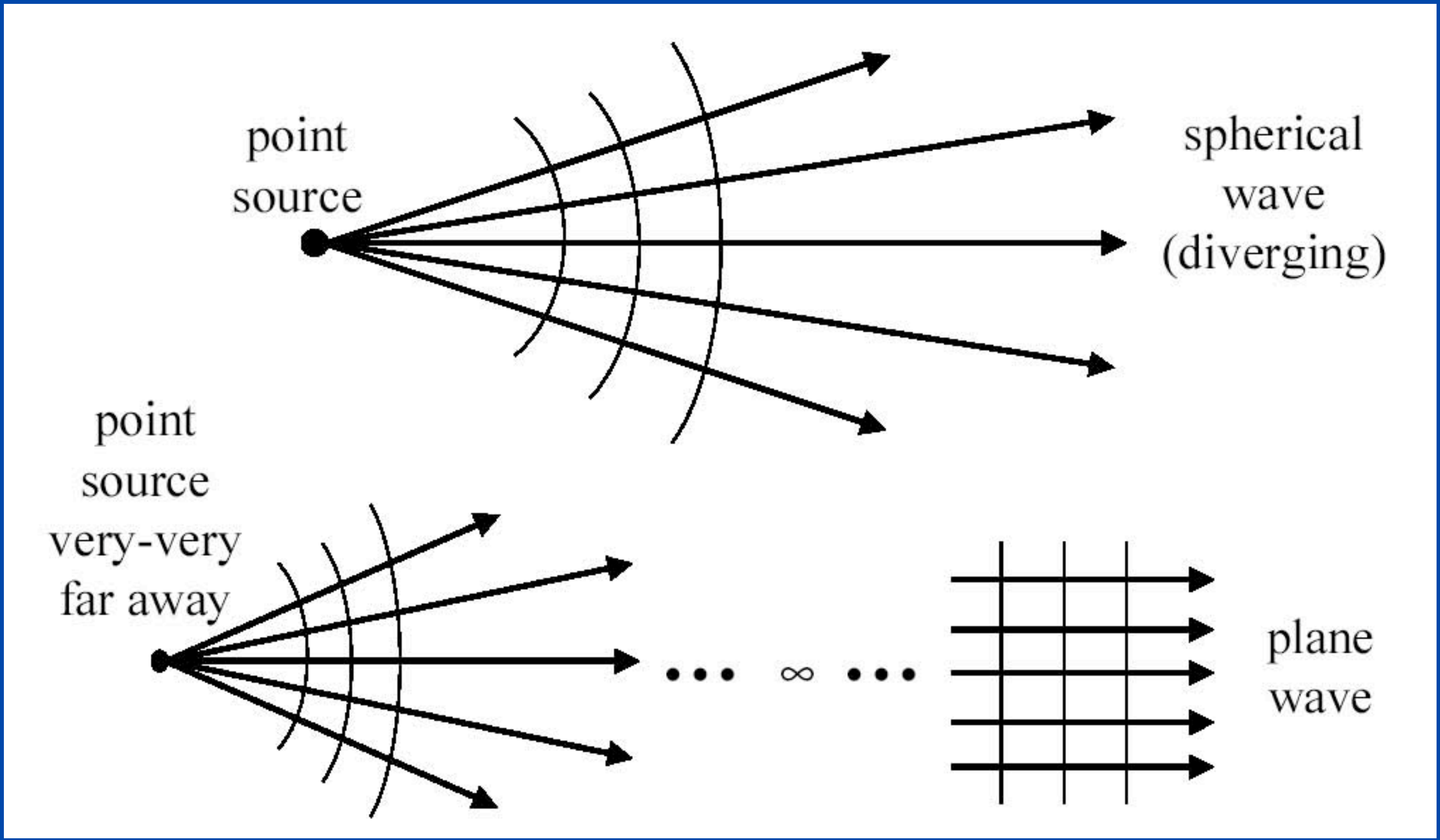


Rays and wavefronts

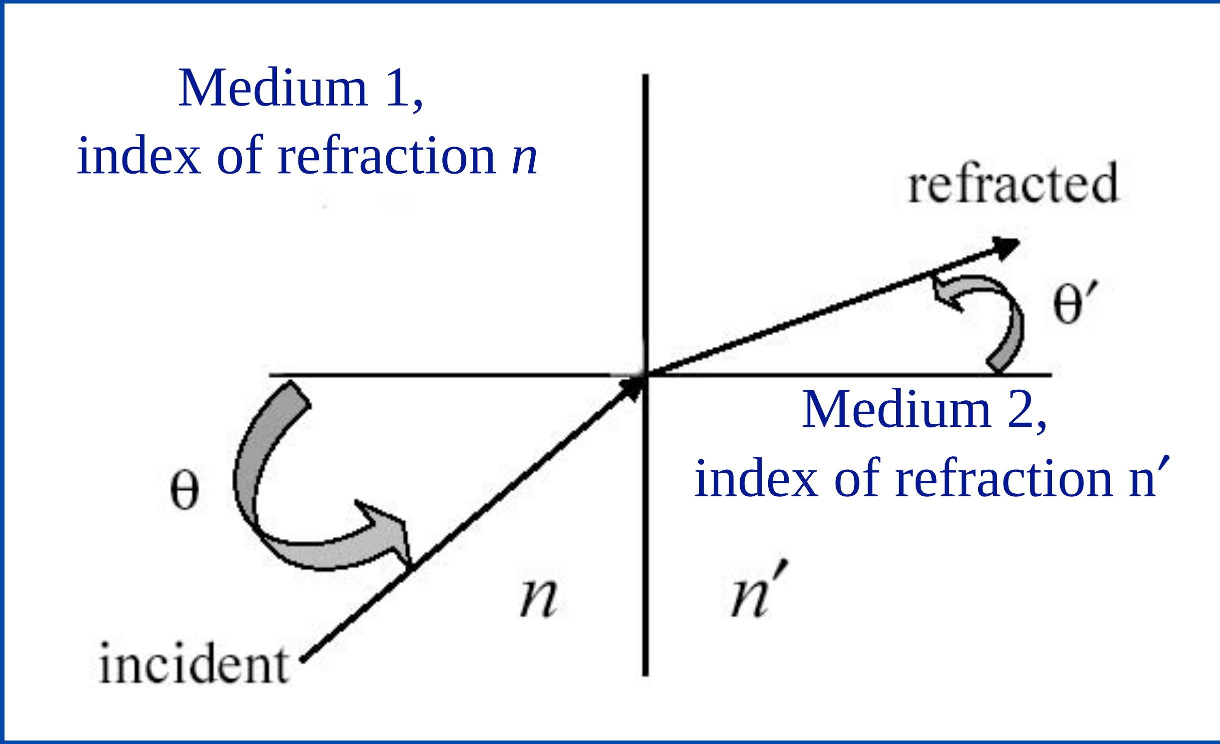


- In homogeneous media, light propagates in straight lines

Spherical waves and plane waves

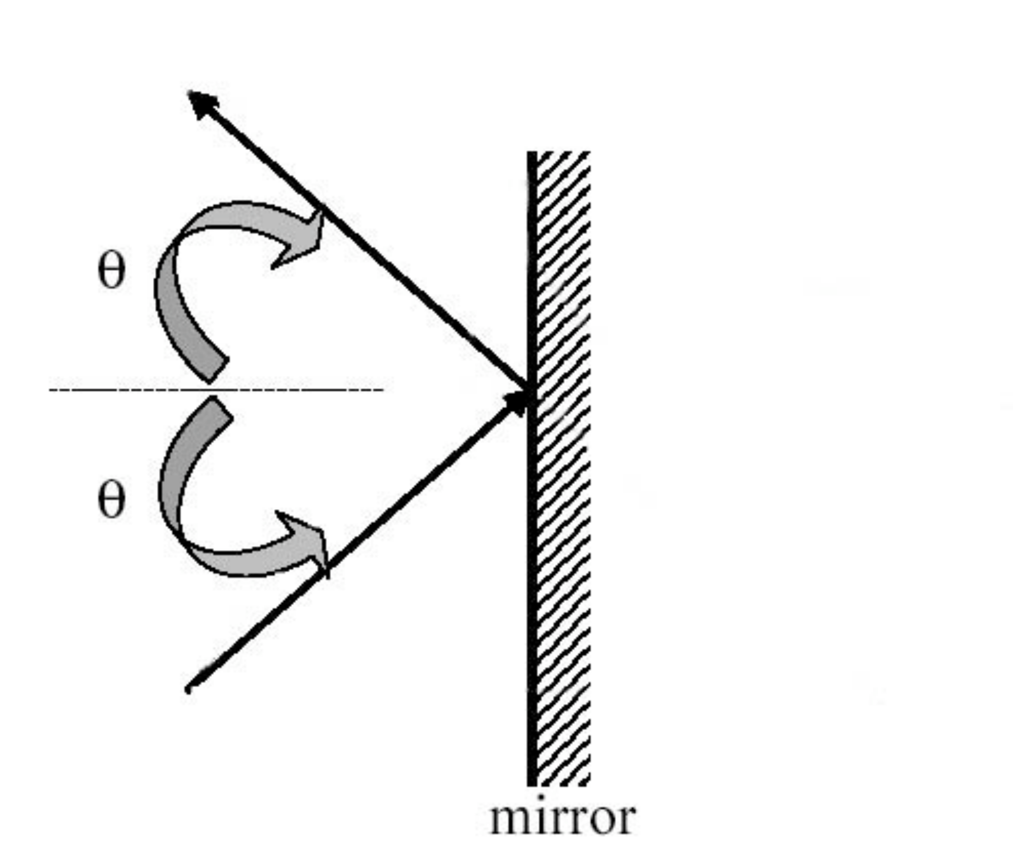


Refraction at a surface: Snell's Law



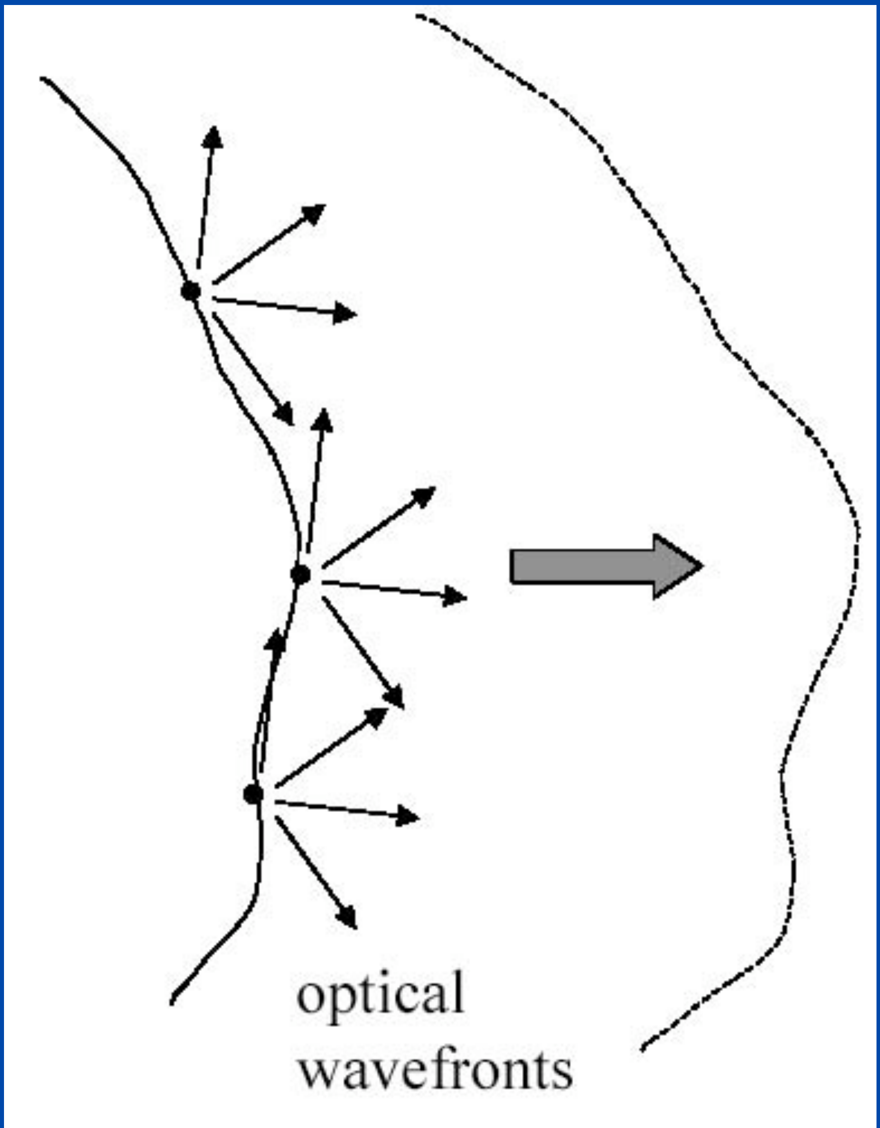
- **Snell's law:** $$n \sin\vartheta = n' \sin\vartheta'$$

Reflection at a surface



- Angle of incidence equals angle of reflection

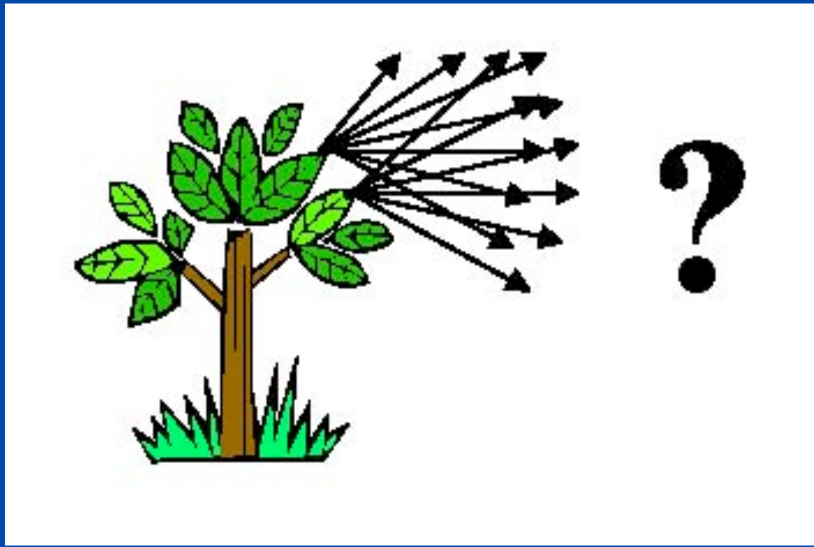
Huygens' Principle



- Every point in a wavefront acts as a little secondary light source, and emits a spherical wave
- The propagating wave-front is the result of superposing all these little spherical waves
- Destructive interference in all but the direction of propagation

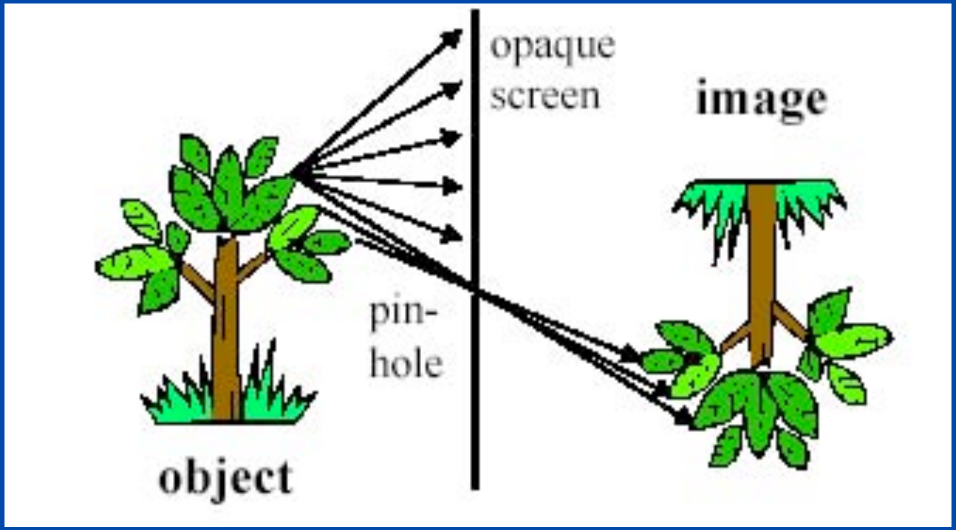
So why are imaging systems needed?

- Every point in the object scatters incident light into a spherical wave
- The spherical waves from all the points on the object's surface get mixed together as they propagate toward you
- An imaging system reassigns (focuses) all the rays from a single point on the object onto another point in space (the "focal point"), so you can distinguish details of the object

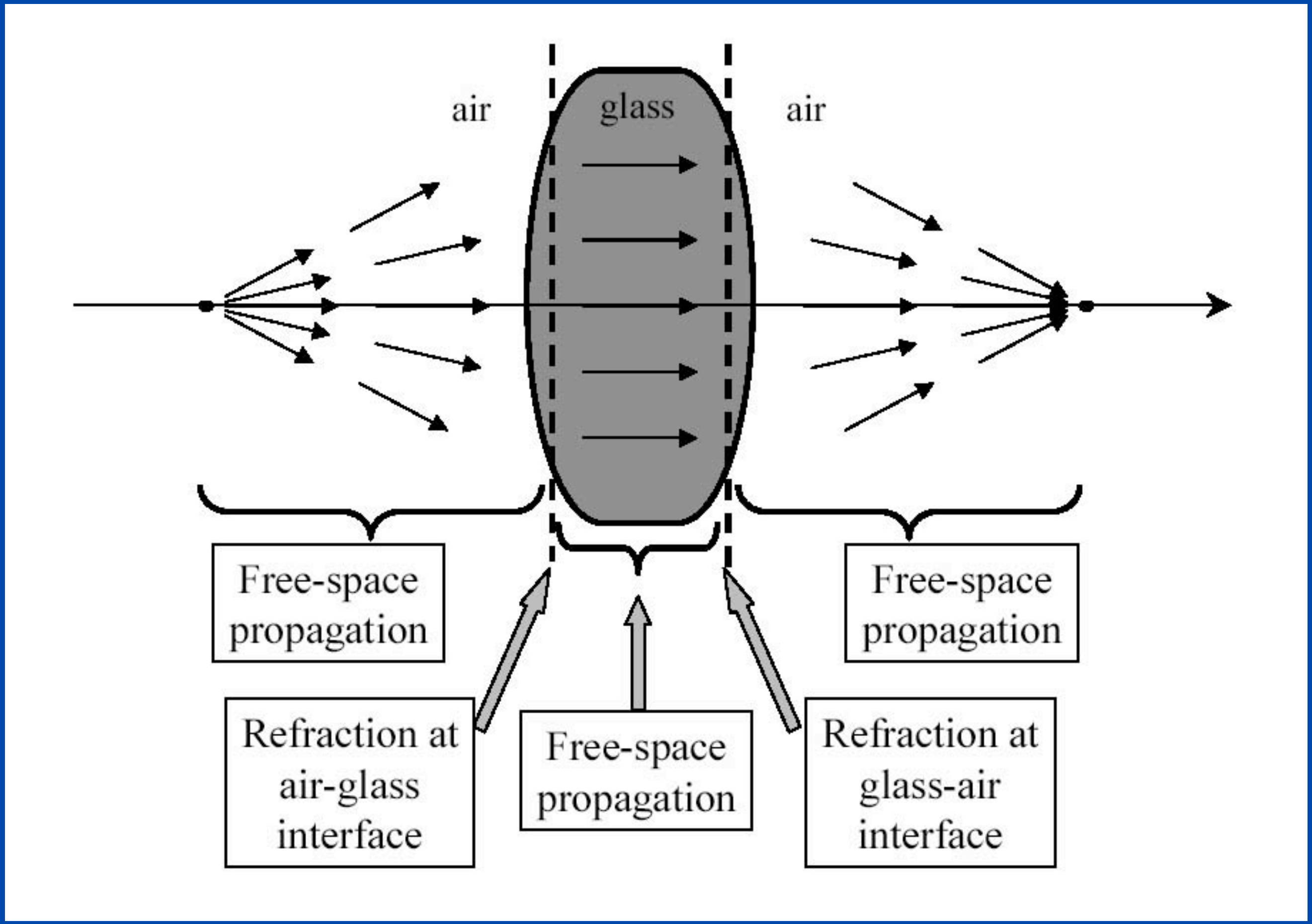


Pinhole camera is simplest imaging instrument

- Opaque screen with pinhole blocks all but one ray per object point from reaching the image space
- An image is formed (upside down)
- BUT most of the light is wasted (it is stopped by the opaque sheet)
- Also, diffraction of light as it passes through the small pinhole produces artifacts in the image



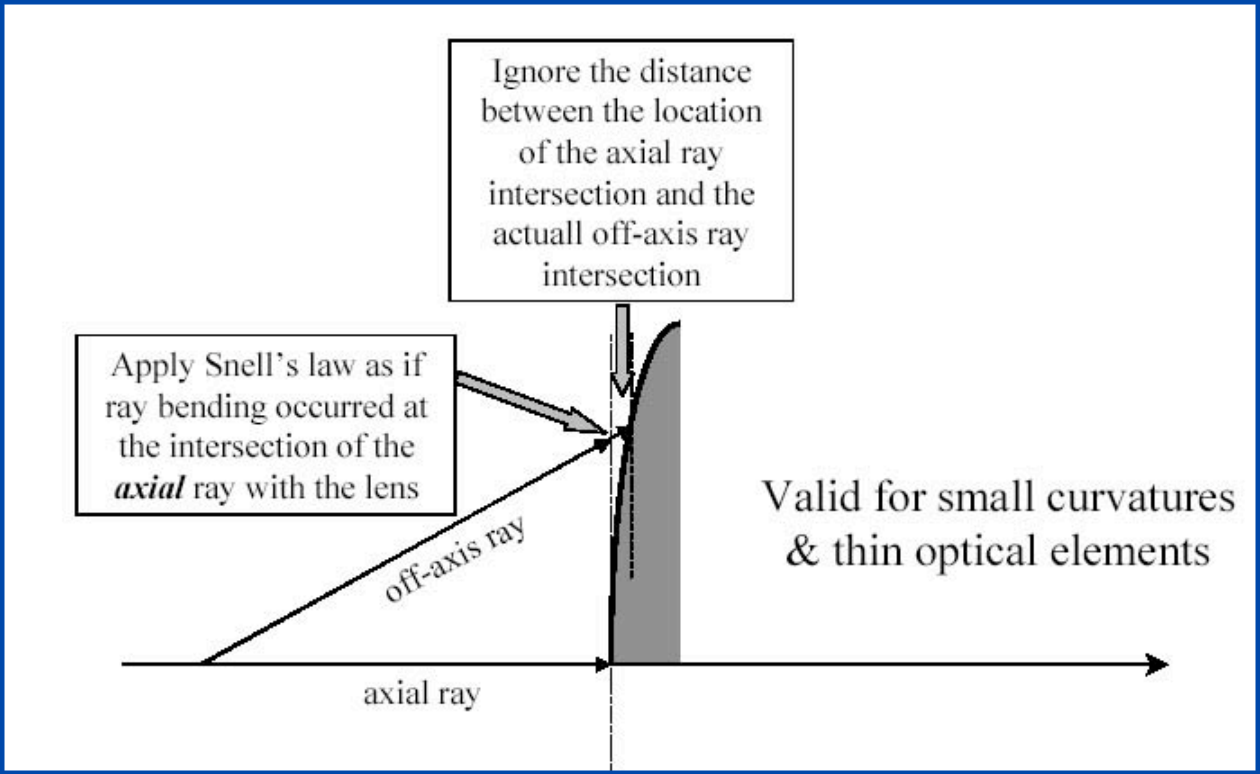
Imaging with lenses: doesn't throw away as much light as pinhole camera



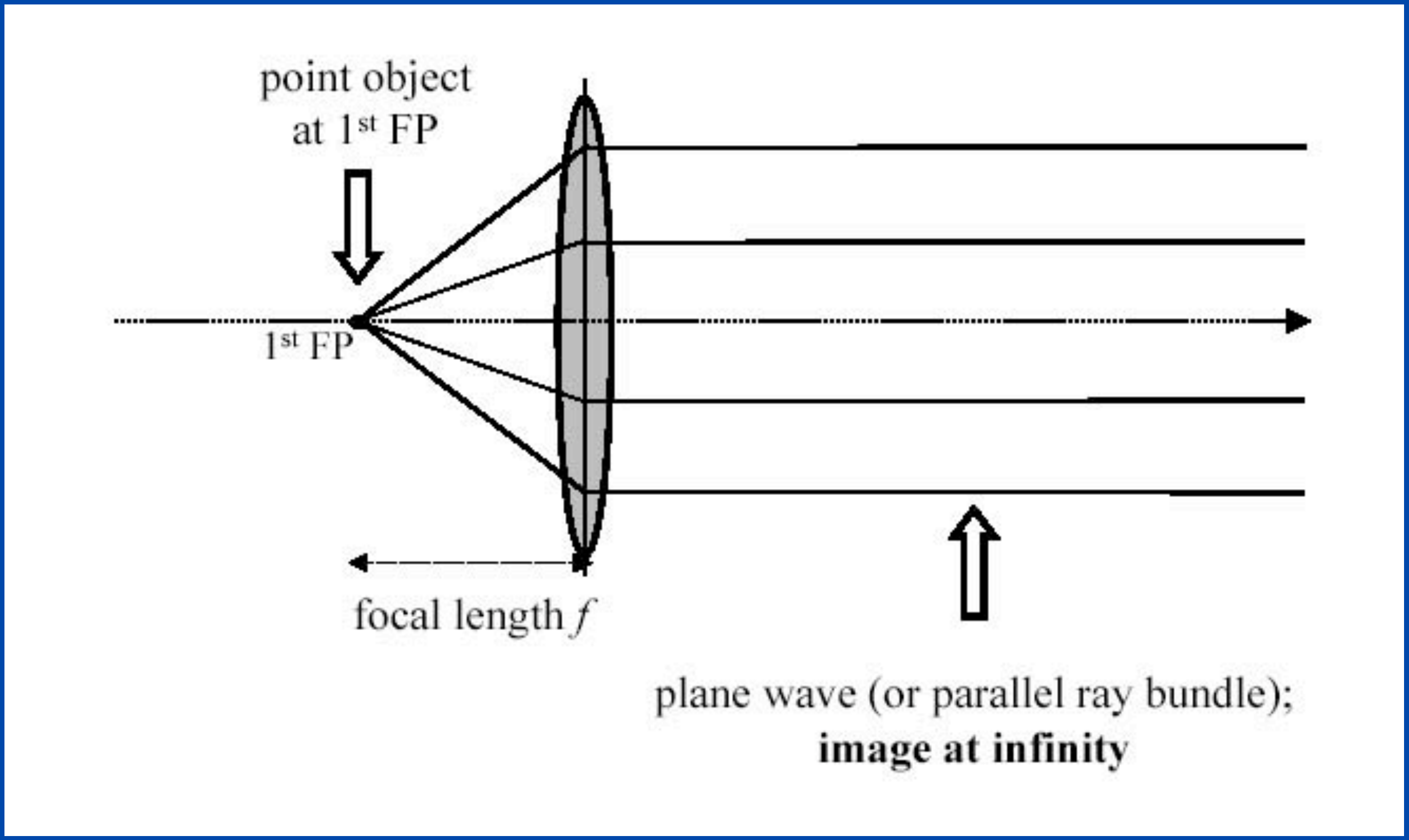
Collects <u>all</u> rays that pass through solid-angle of lens

"Paraxial approximation" or "first order optics" or "Gaussian optics"

- Angle of rays with respect to optical axis is small
- First-order Taylor expansions:
  - $\sin \varepsilon \approx \tan \varepsilon \approx \varepsilon$, $\cos \varepsilon \approx 1$, $(1 + \varepsilon)^{1/2} \approx 1 + \varepsilon / 2$

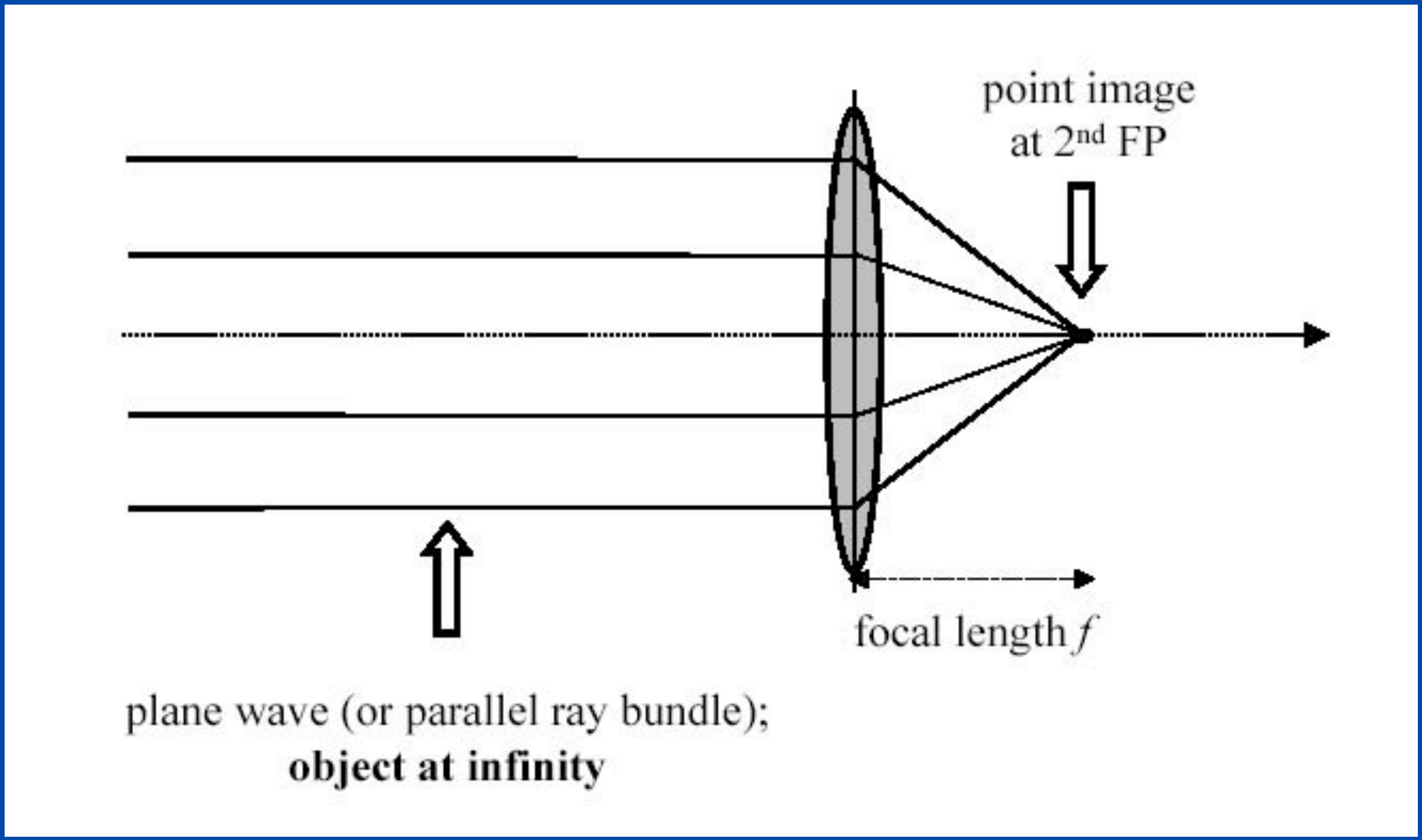


Thin lenses, part 1



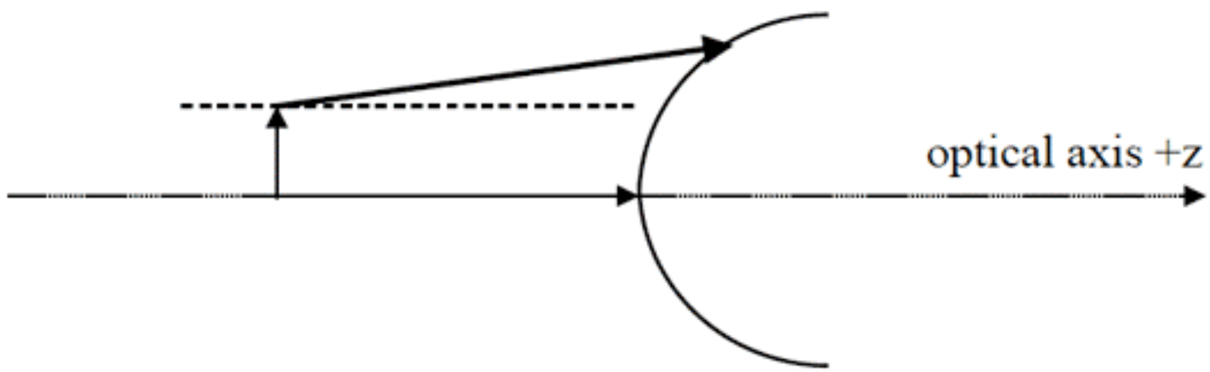
Definition: *f*-number ≡ *f* / # = *f* / *D*

Thin lenses, part 2

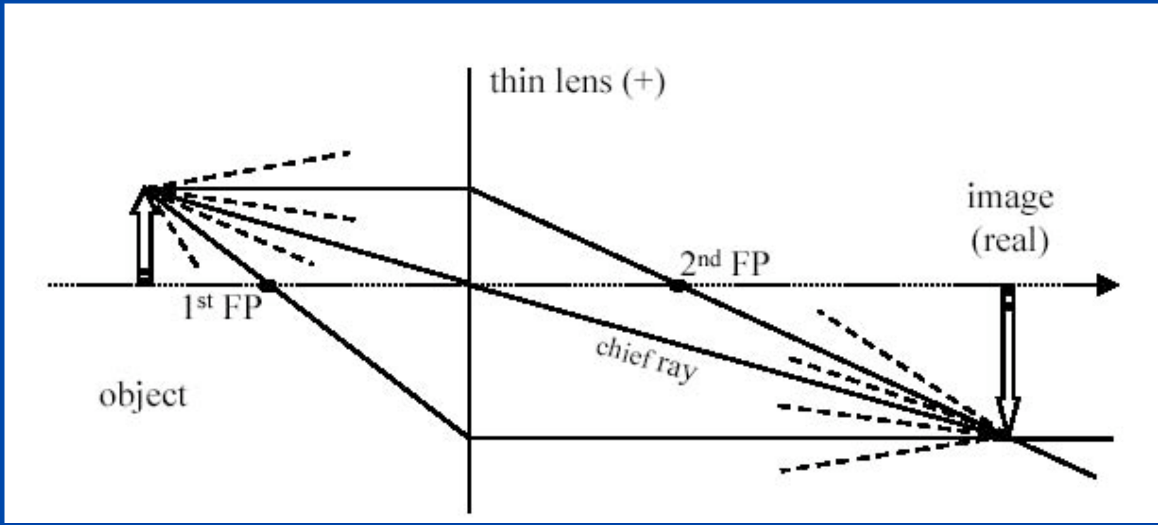


Sign conventions for refraction

- Light travels from left to right
- A radius of curvature is positive if the surface is convex towards the left
- Longitudinal distances are positive if pointing to the right
- Lateral distances are positive if pointing up
- Ray angles are positive if the ray direction is obtained by rotating the +z axis counterclockwise through an acute angle



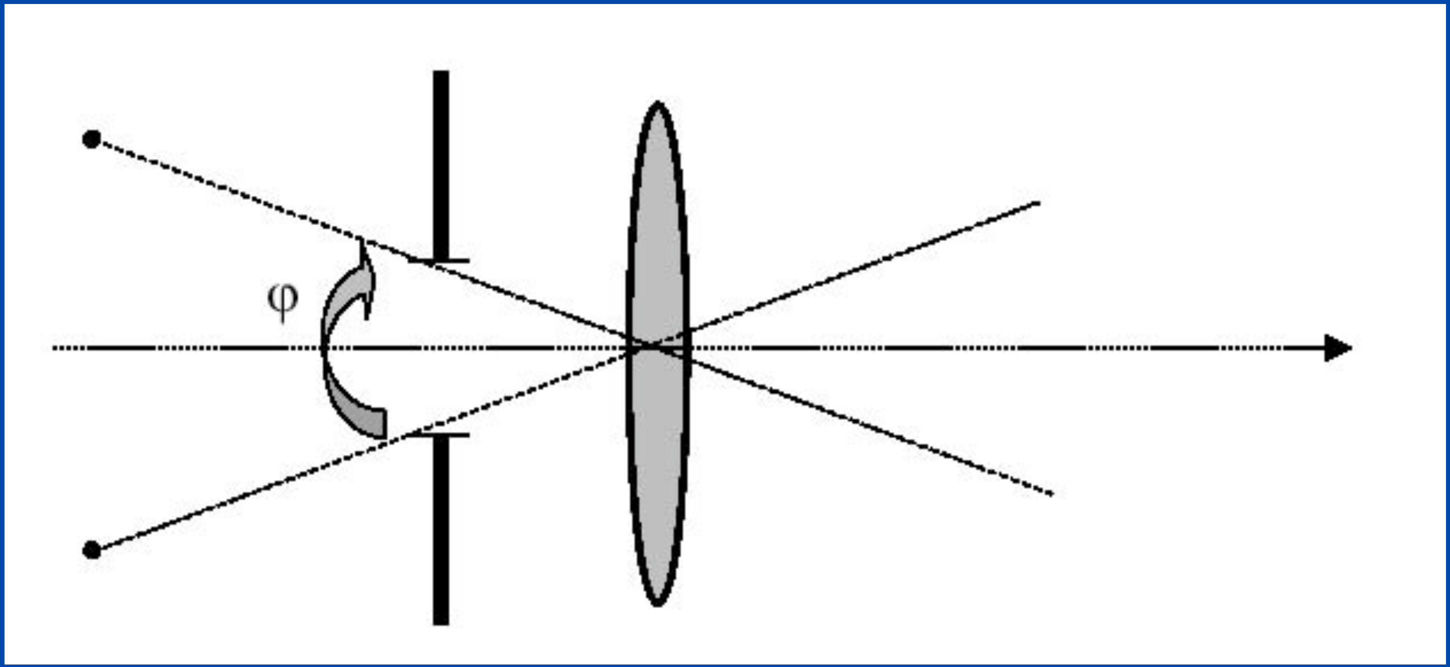
Ray-tracing with a thin lens



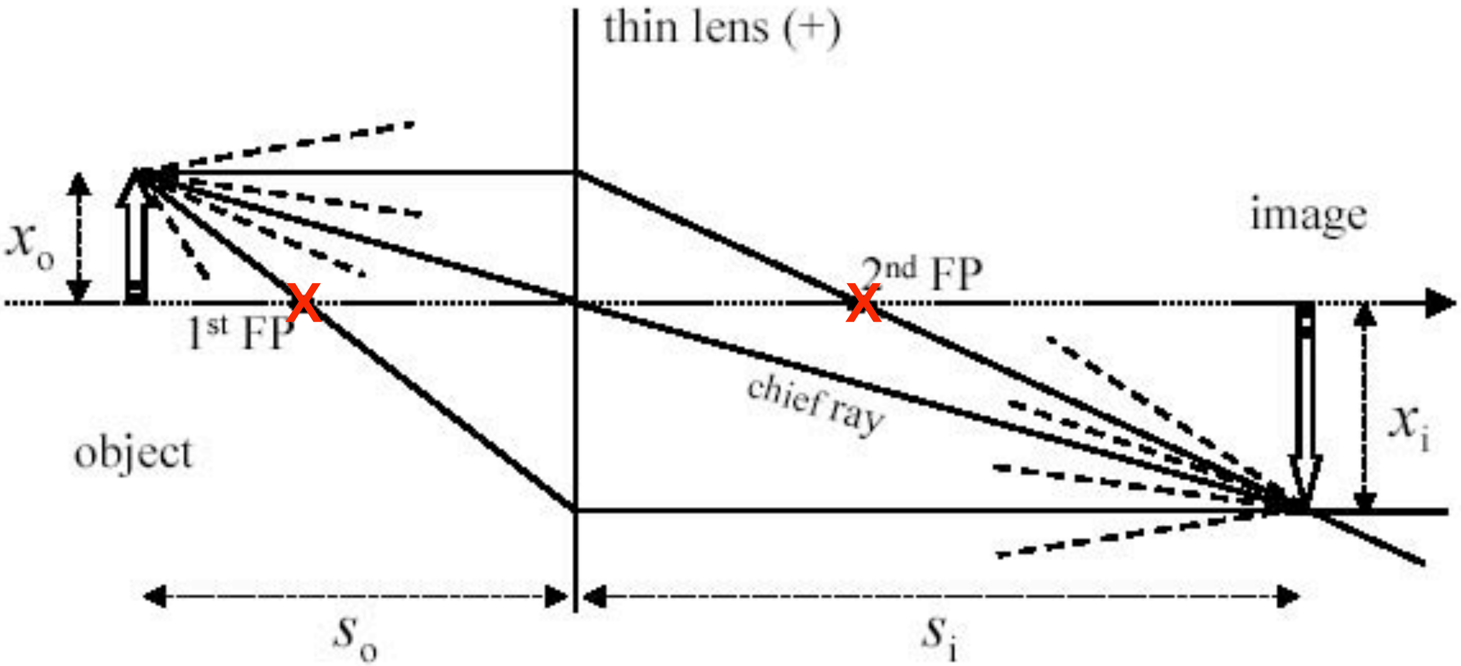
- Image point (focus) is located at intersection of ALL rays passing through the lens from the corresponding object point
- Easiest way to see this: trace rays passing through the two foci, and through the center of the lens (the "chief ray")

Definition: Field of view (FOV) of an imaging system

- Angle that the "chief ray" from an object can subtend, given the pupil (entrance aperture) of the imaging system
- Recall that the chief ray propagates through the lens undeviated



Summary of important relationships for lenses



Lens Law

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

Lateral magnification

$$M_x = \frac{x_i}{x_o} = -\frac{s_o}{s_i}$$

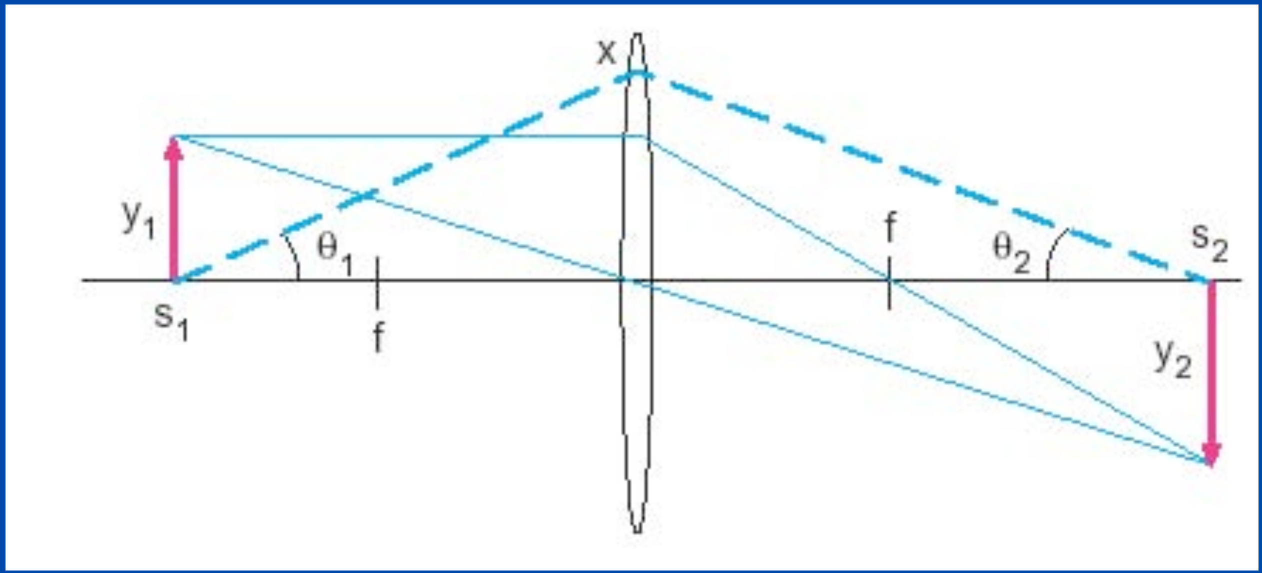
Angular magnification

$$M_a = -\frac{s_i}{s_o}$$

Energy conservation

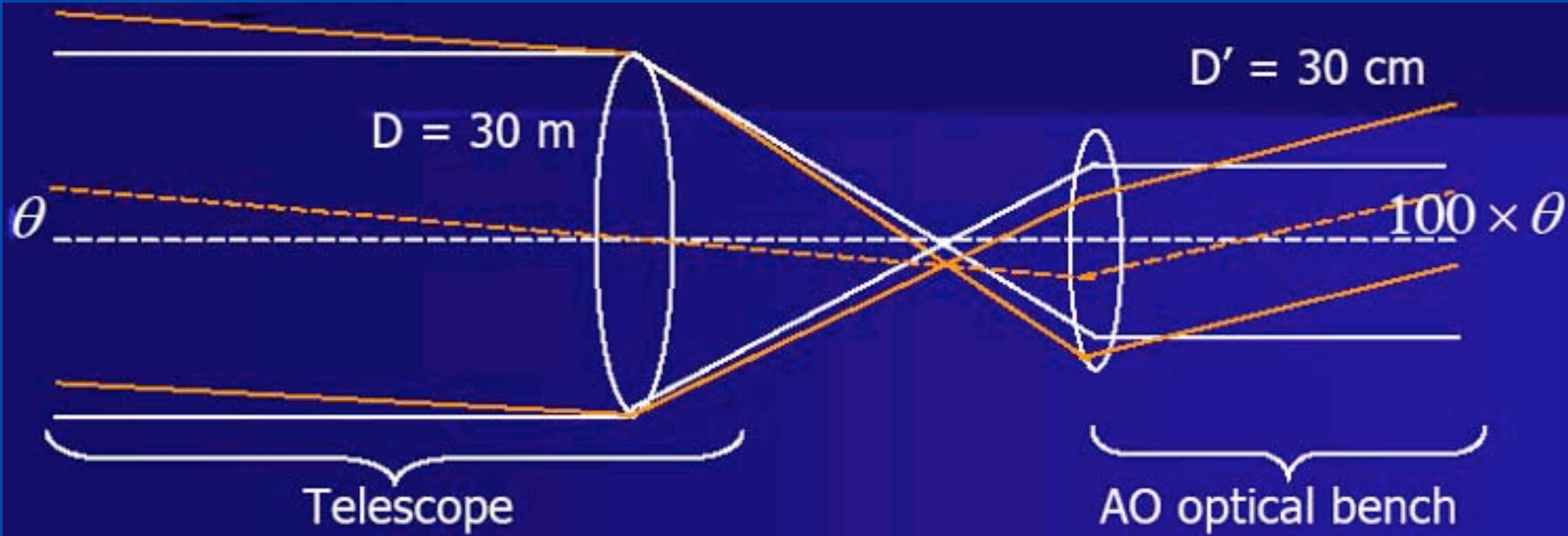
$$M_x M_a = 1$$

Optical invariant (= Lagrange invariant)



$$y_1\vartheta_1 = y_2\vartheta_2$$

Lagrange invariant has important consequences for AO on large telescopes



M is transverse magnification (10^{-2})

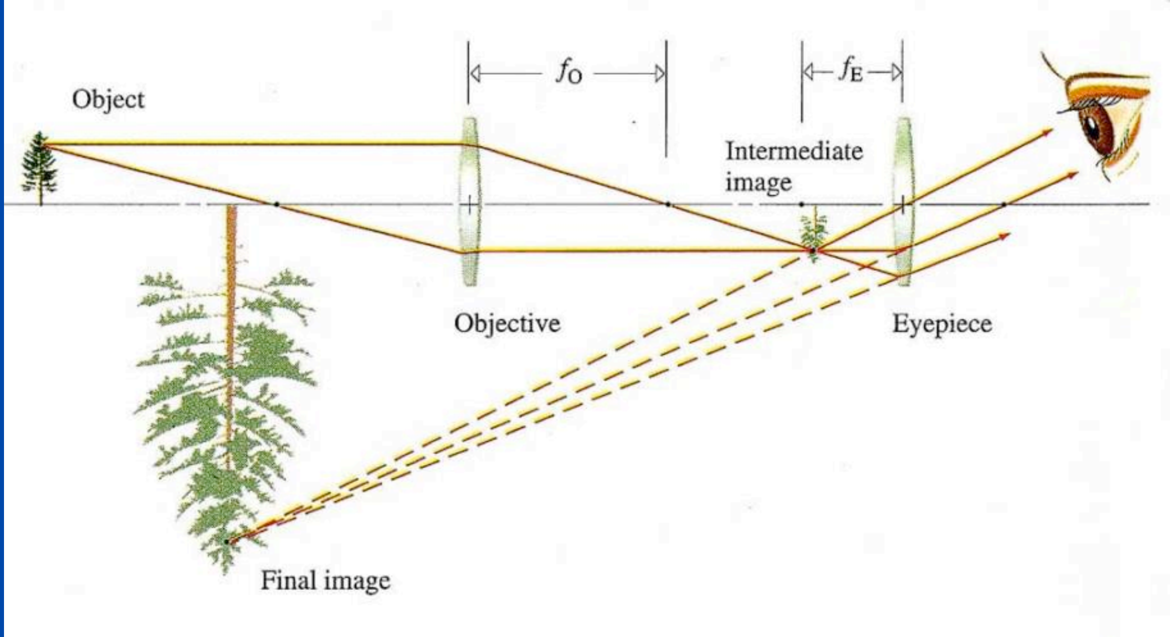
$$D' = MD \qquad L' = M^2 L \qquad \theta' = \frac{D'}{L'} = M^{-1}\theta$$

Preserves the *Lagrange invariant*:

$$\Xi = \theta' D' = \theta D$$

From Don Gavel

Refracting telescope

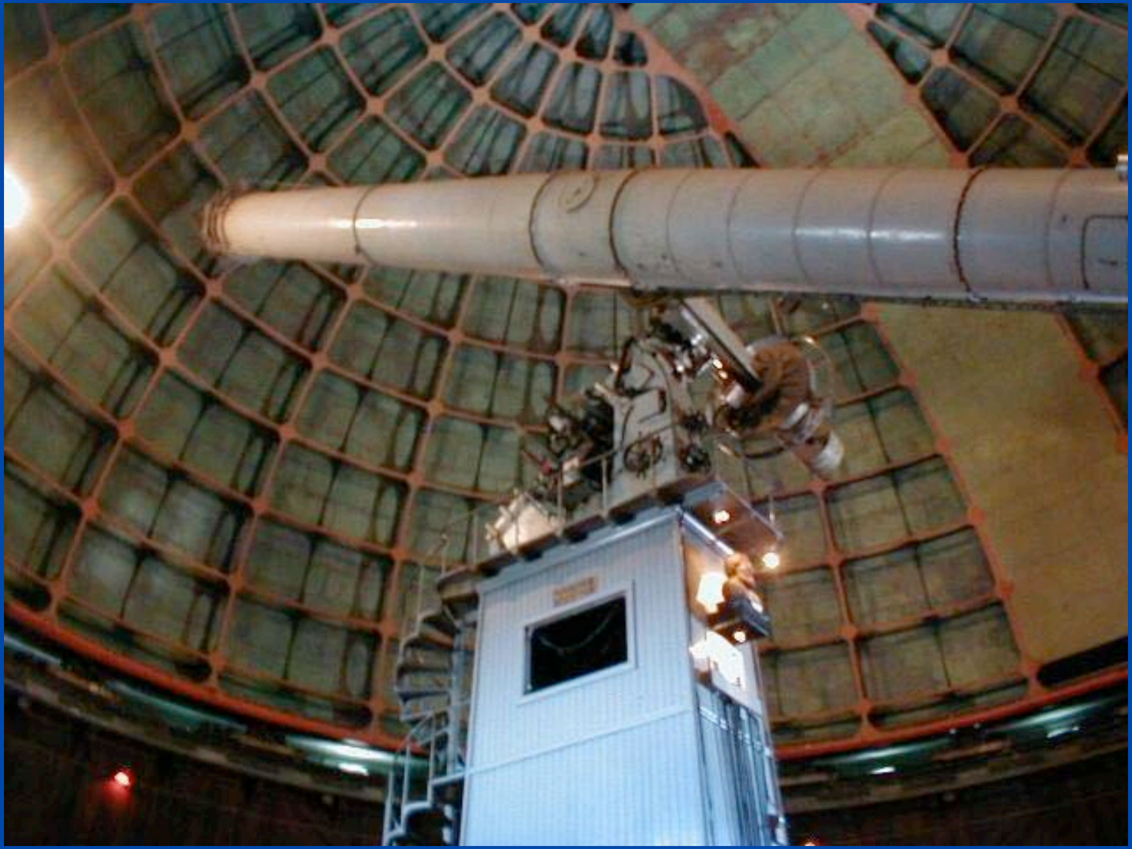


$$\frac{1}{f_{obj}} = \frac{1}{s_0} + \frac{1}{s_1} \approx \frac{1}{s_1} \text{ since } s_0 \to \infty$$

$$\text{so } s_1 \approx f_{obj}$$

- **Main point of telescope: to gather more light than eye. Secondarily, to magnify image of the object**
- **Magnifying power $M_{tot} = - f_{Objective} / f_{Eyepiece}$ so for high magnification, make $f_{Objective}$ as large as possible (long tube) and make $f_{Eyepiece}$ as short as possible**

Lick Observatory's 36" Refractor: one long telescope!



Concept Question

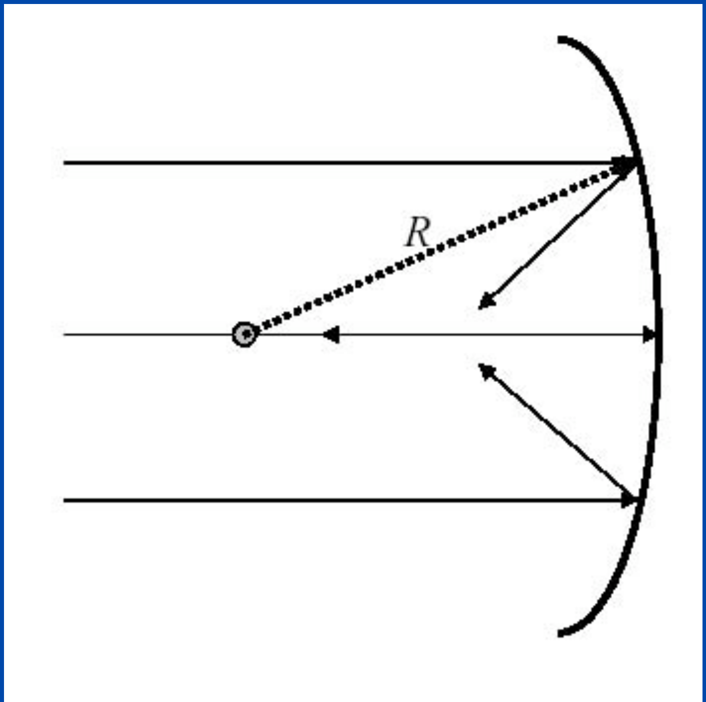
- Give an intuitive explanation for why the magnifying power of a refracting telescope is

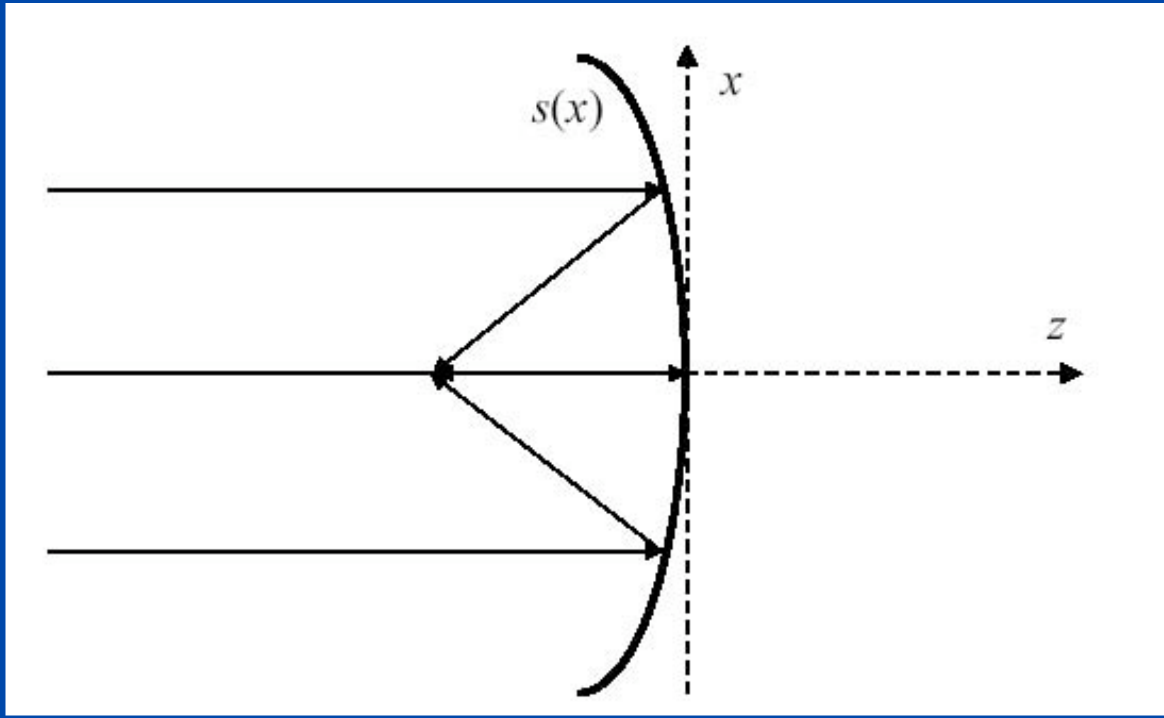
$$M_{tot} = - f_{Objective} / f_{Eyepiece}$$

Make sketches to illustrate your reasoning

Imaging with mirrors: spherical and parabolic mirrors

$f = R/2$





<u>Spherical surface</u>: in paraxial approx, focuses incoming parallel rays to (approx) a point

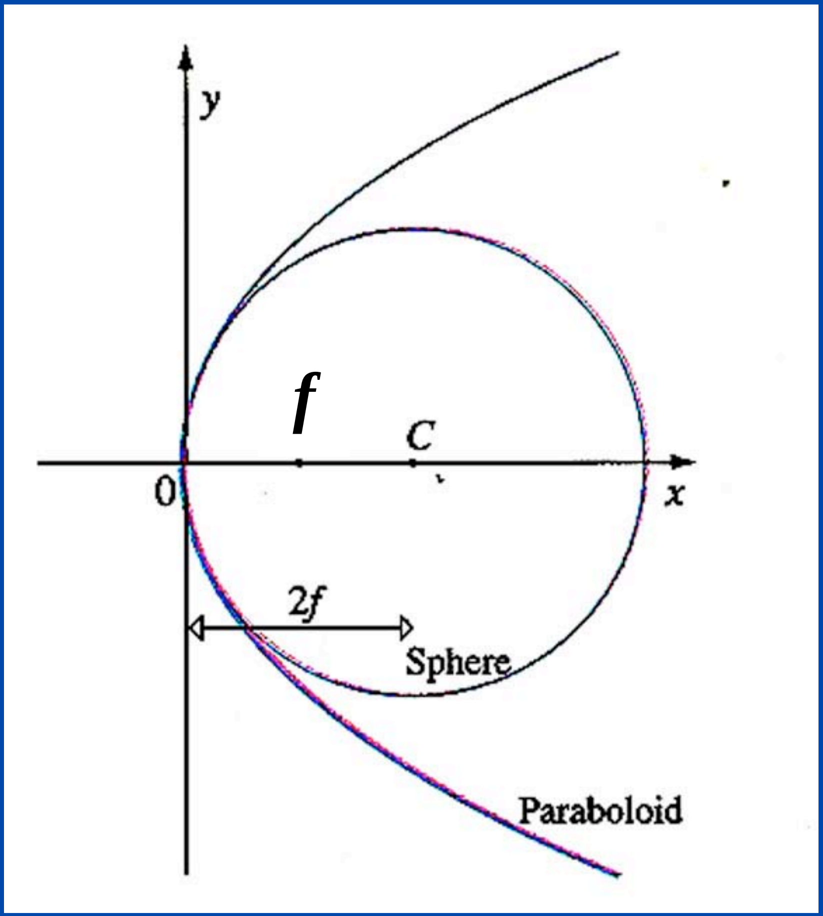
<u>Parabolic surface</u>: perfect focusing for parallel rays (e.g. satellite dish, radio telescope)

Problems with spherical mirrors

- Optical aberrations (mostly spherical and coma), especially if *f*-number is small ("fast" focal ratio)

Focal length of mirrors

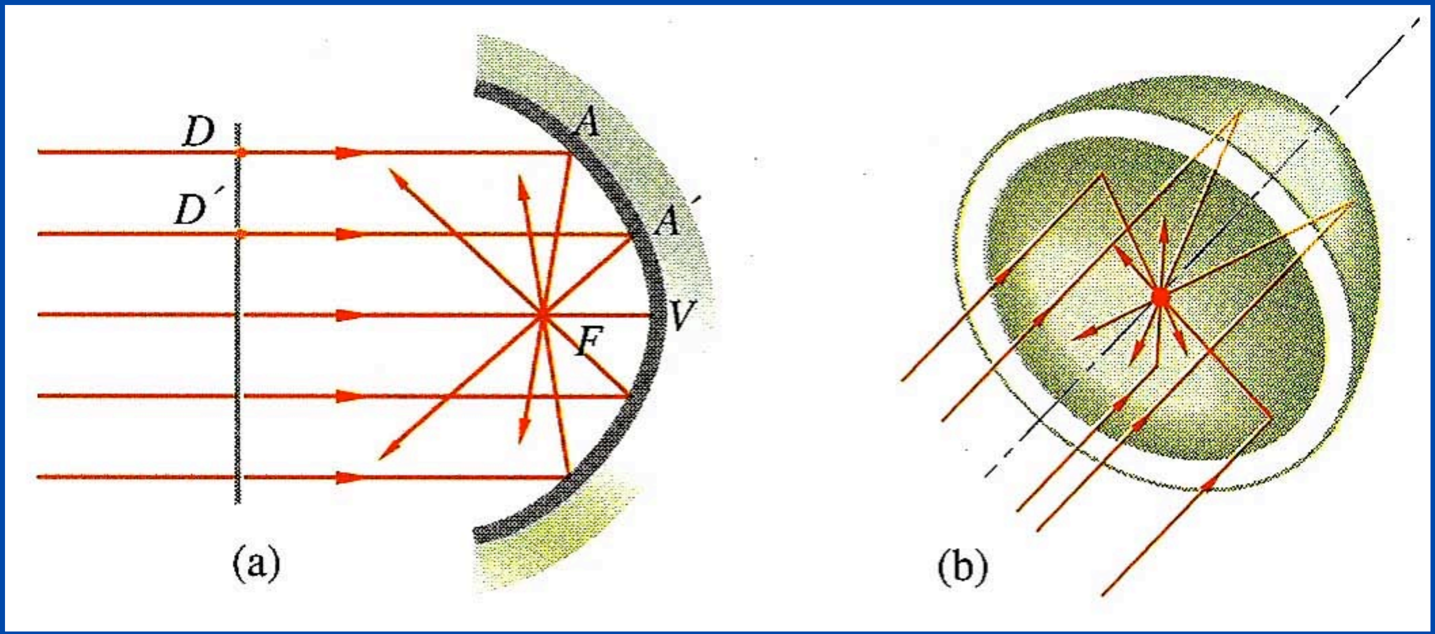
- Focal length of spherical mirror is $f_{sp} = -R/2$
- Convention: f is positive if it is to the left of the mirror
- Near the optical axis, parabola and sphere are very similar, so that $f_{par} = -R/2$ as well.



Sign conventions for reflection

- Light travels from left to right *before reflection and from right to left after reflection*
- A radius of curvature is positive if the surface is convex towards the left
- Longitudinal distances *before reflection* are positive if pointing to the right; *longitudinal distances after reflection are positive if pointing to the left*
- Longitudinal distances are positive if pointing up
- Ray angles are positive if the ray direction is obtained by rotating the +z axis counterclockwise through an acute angle

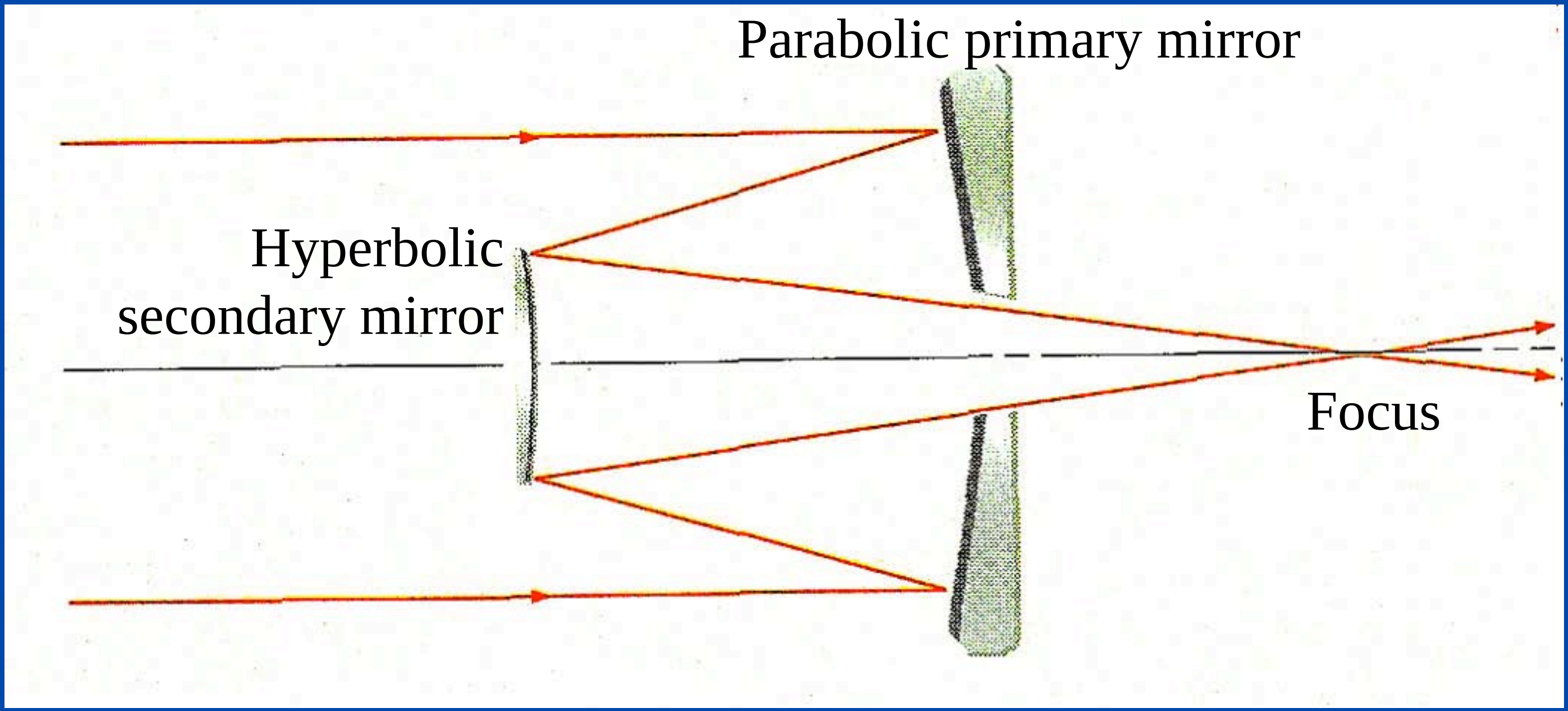
Parabolic mirror: focus in 3D



Mirror formulae

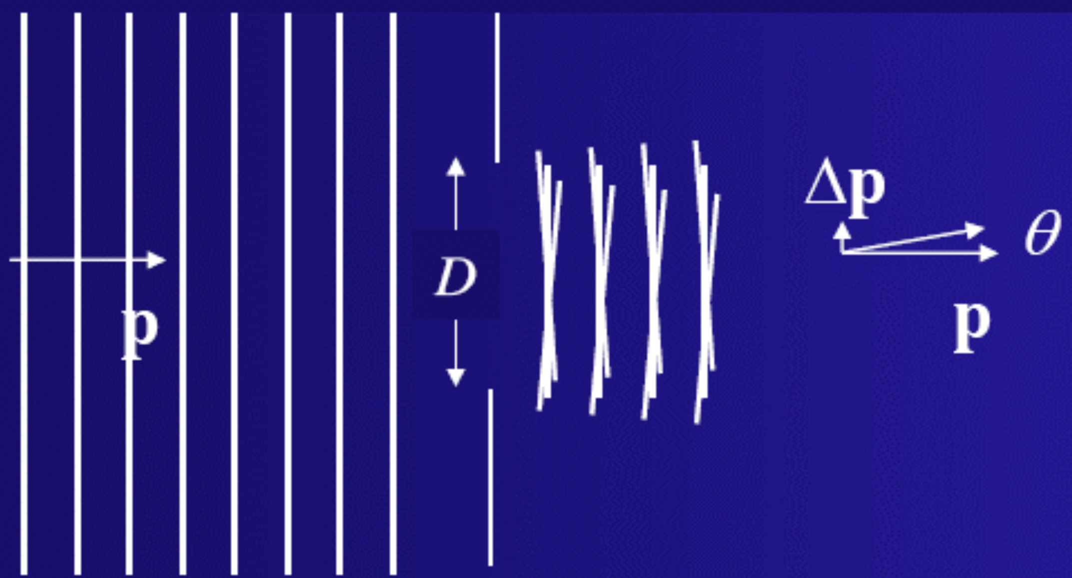
- Imaging condition for spherical mirror $$\frac{1}{s_0} + \frac{1}{s_1} = -\frac{2}{R}$$
- Focal length $$f = -\frac{R}{2}$$
- Magnifications $$M_{transverse} = -\frac{s_0}{s_1}$$ $$M_{angle} = -\frac{s_1}{s_0}$$

Cassegrain reflecting telescope



- Hyperbolic secondary mirror: 1) reduces off-axis aberrations, 2) shortens physical length of telescope.
- Can build mirrors with much shorter focal lengths than lenses. Example: 10-meter primary mirrors of Keck Telescopes have focal lengths of 17.5 meters (f/1.75). About same as Lick 36” refractor.

Heuristic derivation of the diffraction limit



Uncertainty principle

$$\Delta x \Delta p \simeq h$$

Photon momentum

$$p = h/\lambda$$

$$\Delta \mathbf{x} = \infty$$

$$\Delta \mathbf{p} = 0$$

$$\Delta x = D$$

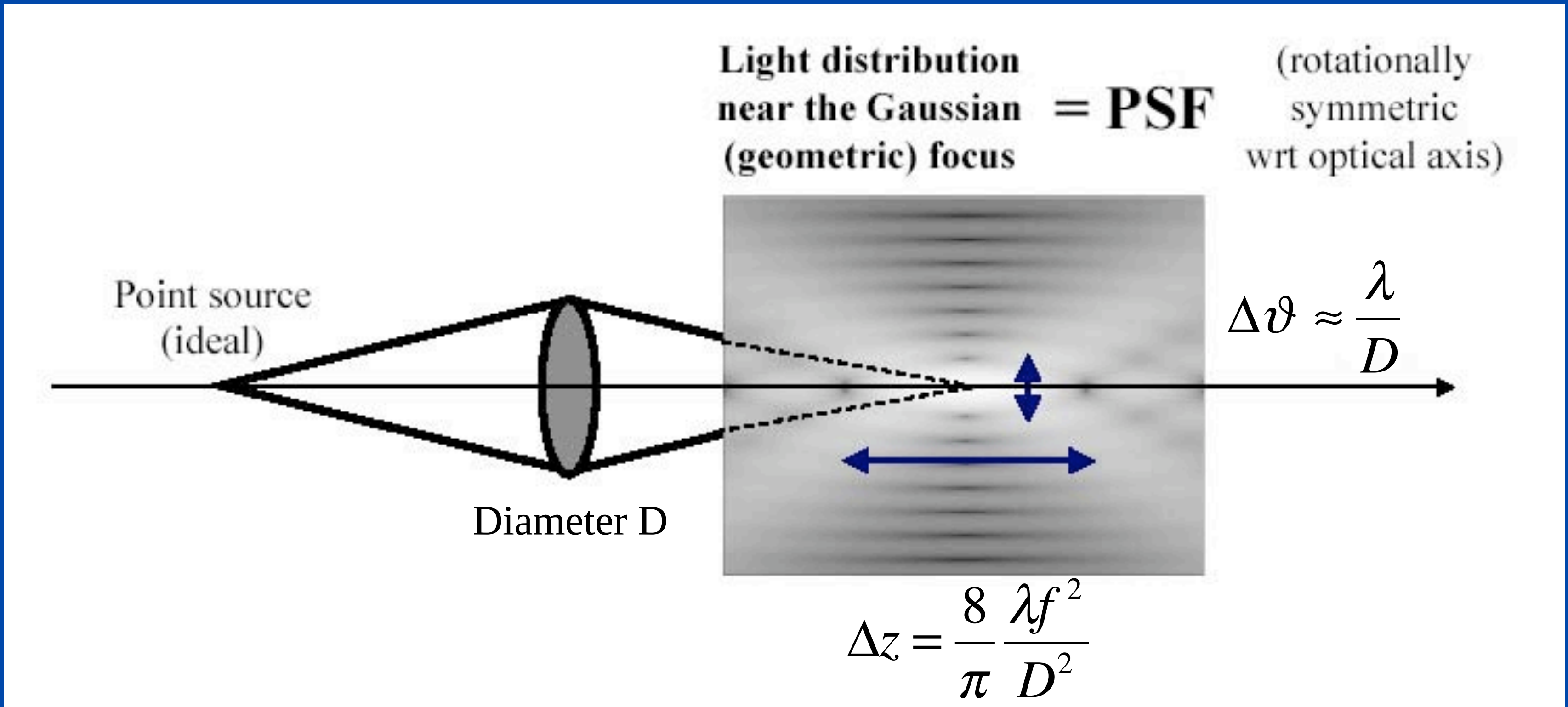
$$\Delta p = h/D$$

$\Rightarrow$ Law of diffraction

$$\theta \simeq \Delta p/p = \lambda/D$$

Courtesy of Don Gavel

Angular resolution and depth of field



- Diffractive calculation ⇒ light doesn't focus at a point. "Beam Waist" has angular width λ / D, and length Δz (depth of field)

Aberrations

- In optical systems
- In atmosphere
- Description in terms of Zernike polynomials

- Based on slides by Brian Bauman, LLNL and UCSC, and Gary Chanan, UCI

Third order aberrations

- *sin θ* terms in Snell's law can be expanded in power series

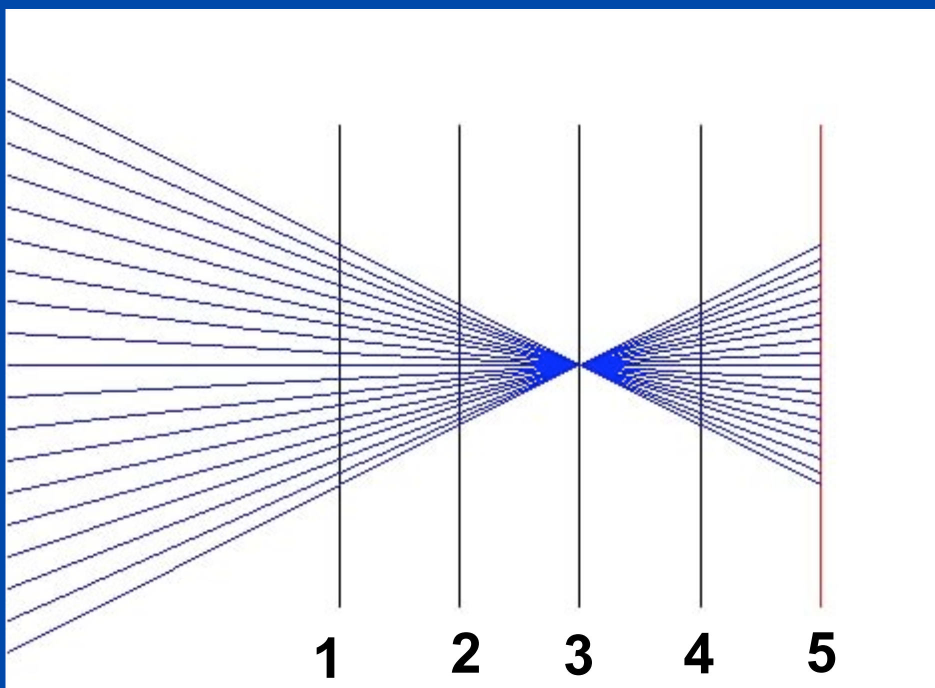
$$n \sin\theta = n' \sin\theta'$$

$$n\,(\theta - \theta^3/3! + \theta^5/5! + \ldots) = n'\,(\theta' - \theta'^3/3! + \theta'^5/5! + \ldots)$$

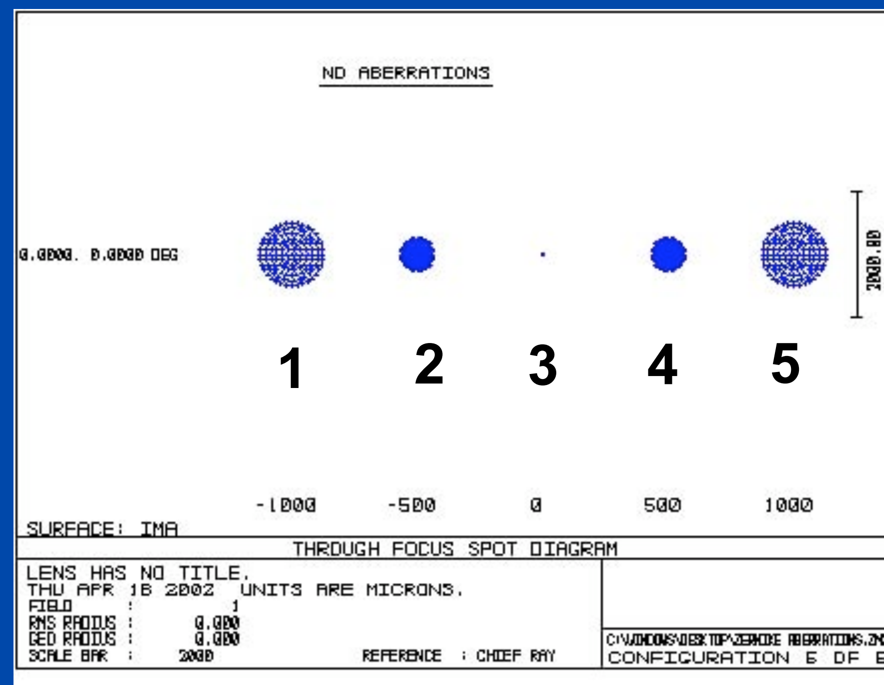
- Paraxial ray approximation: keep only θ terms (first order optics; rays propagate nearly along optical axis)
  - Piston, tilt, defocus
- Third order aberrations: result from adding θ^3 terms
  - Spherical aberration, coma, astigmatism, ...

Different ways to illustrate optical aberrations

Side view of a fan of rays (No aberrations)

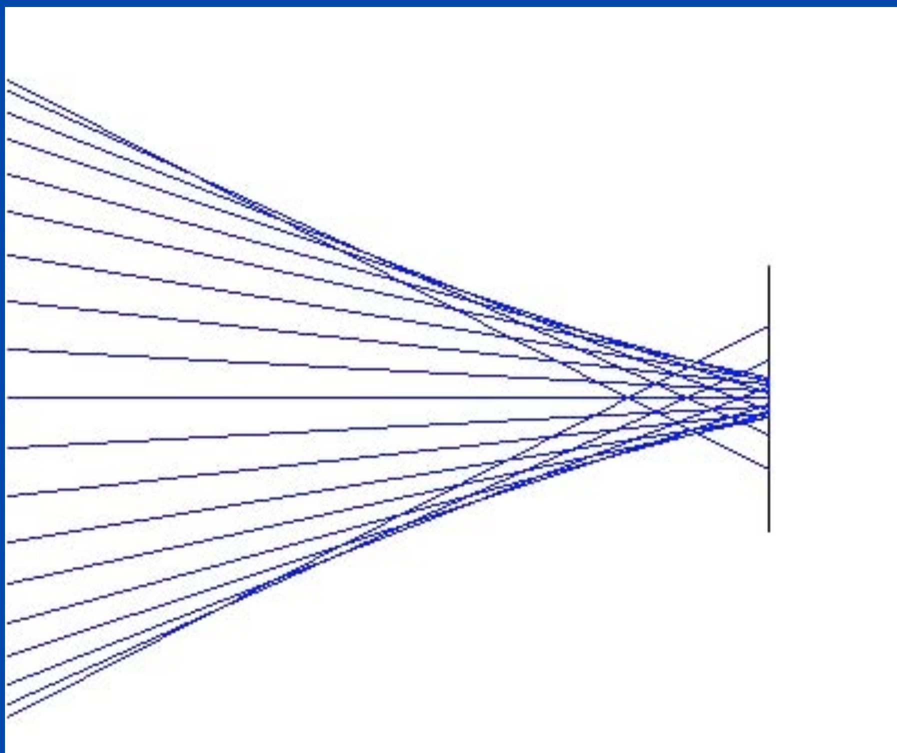


"Spot diagram": Image at different focus positions

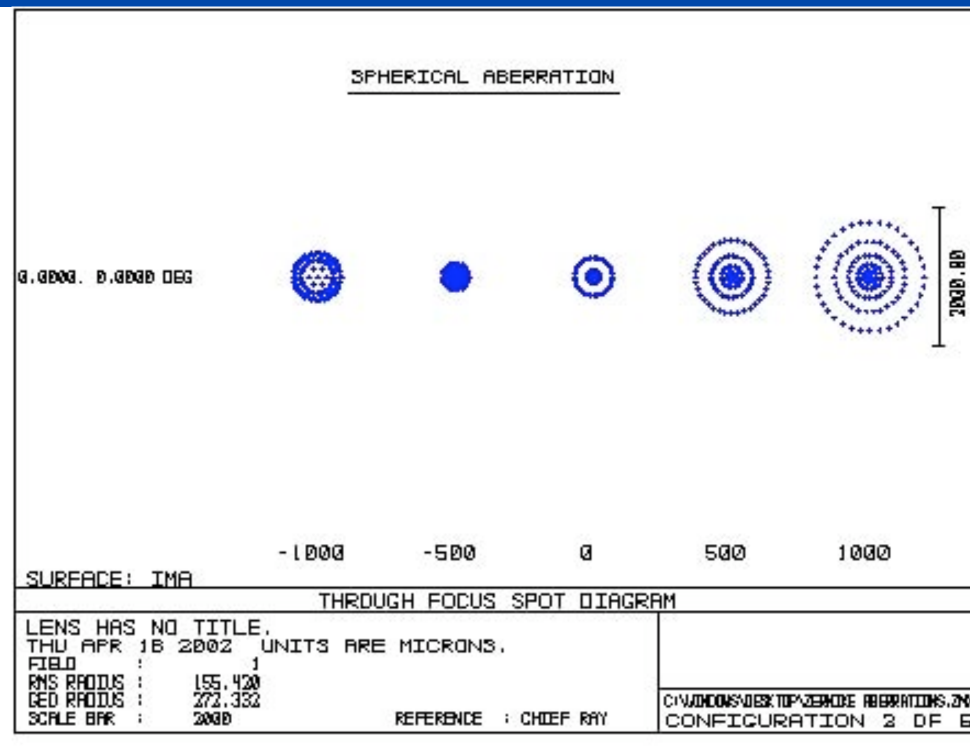


Shows "spots" where rays would strike a detector

Spherical aberration

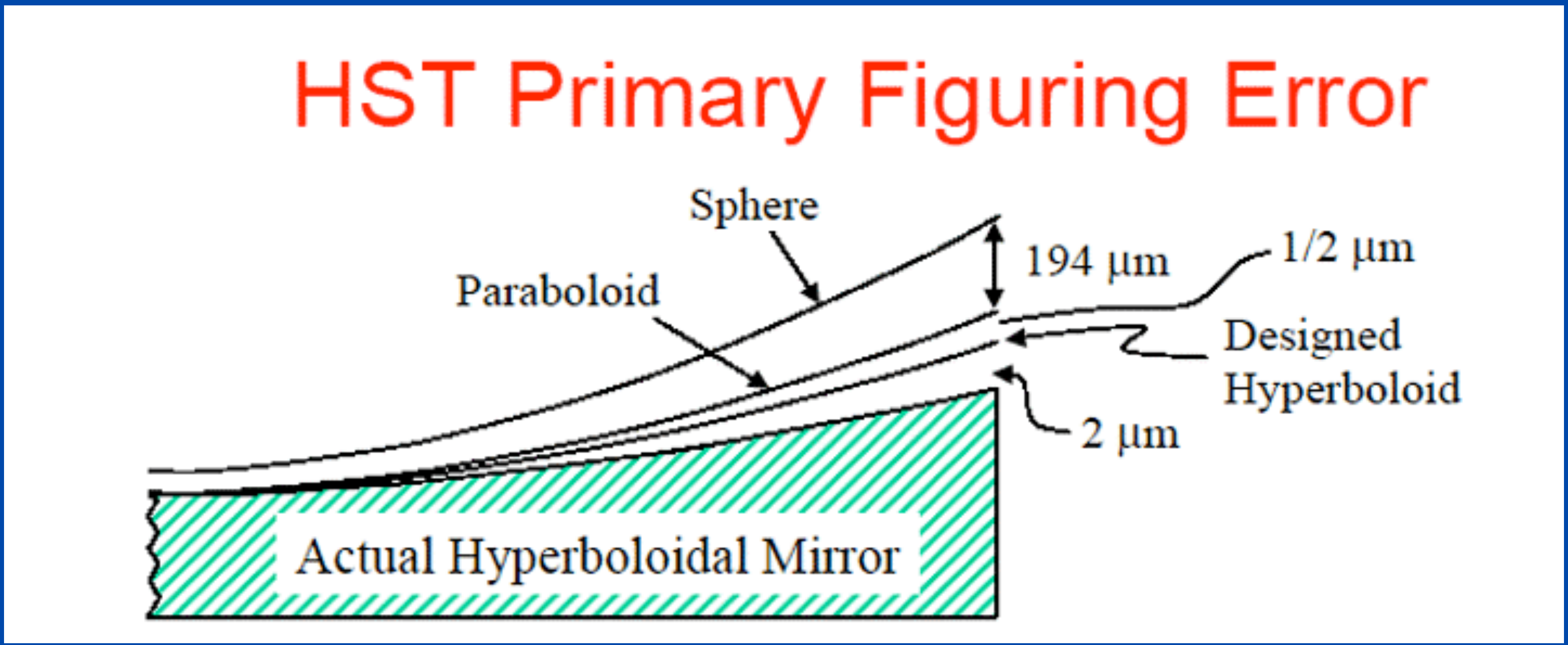


Rays from a spherically aberrated wavefront focus at different planes



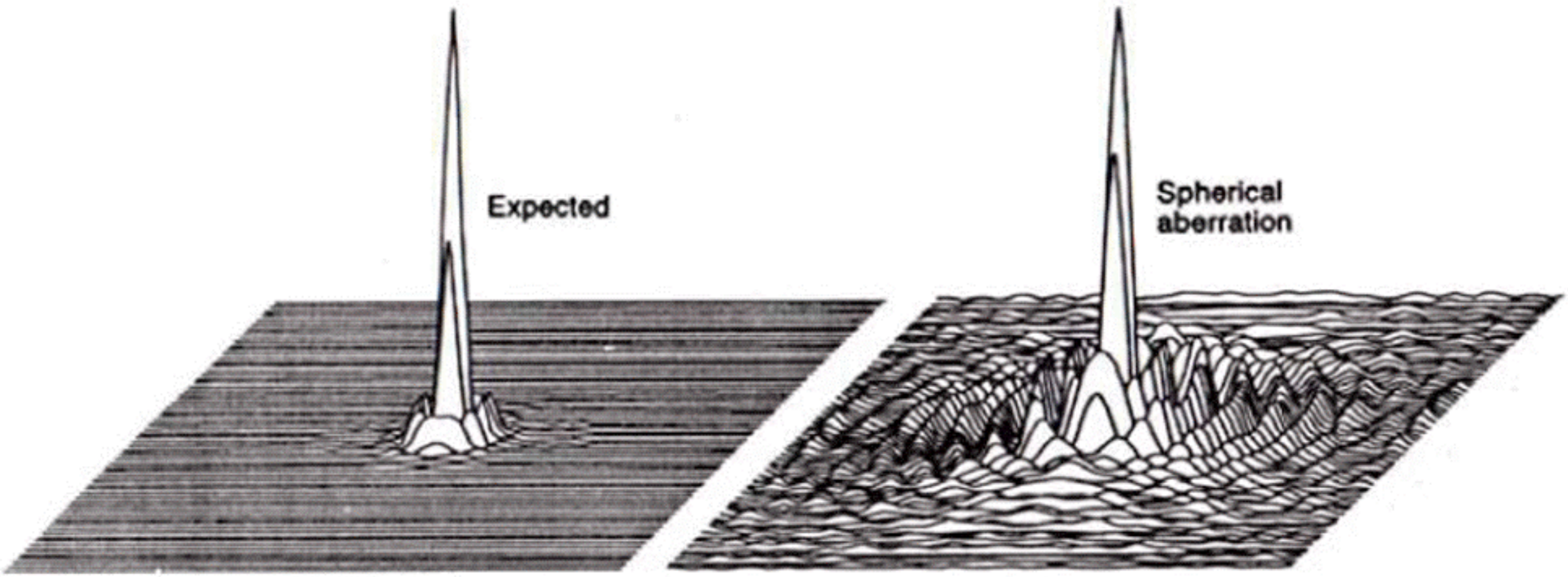
Through-focus spot diagram for spherical aberration

Hubble Space Telescope suffered from Spherical Aberration



- **In a Cassegrain telescope, the hyperboloid of the primary mirror must match the specs of the secondary mirror. For HST they didn't match.**

HST Point Spread Function plots

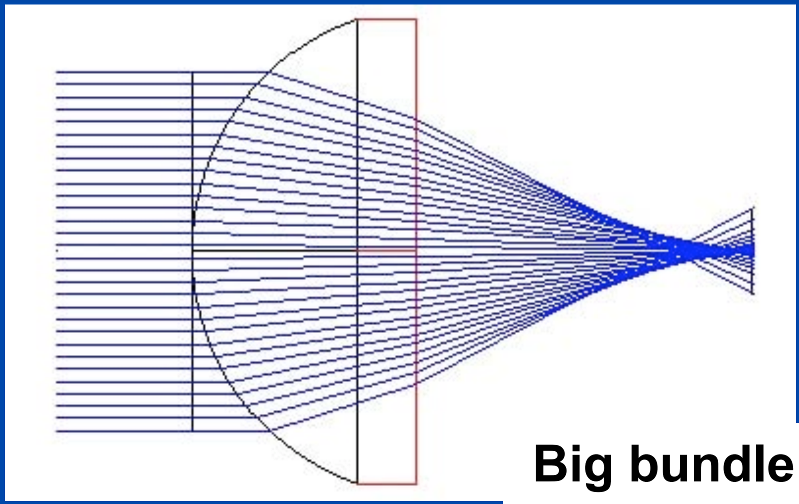


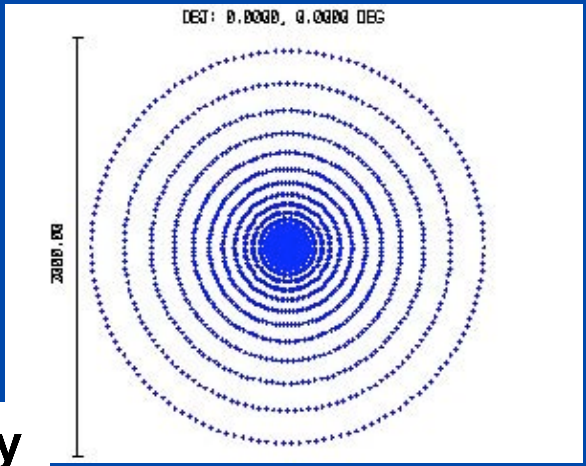
Profiles of HST f/30 planetary camera normalized to the same peak brightness for λ = 0.57 μm. The FWHM of the core is 0.1" in both cases, but only 15% is contained in the spherically aberrated image core.

Spherical aberration as "the parent of all other aberrations"

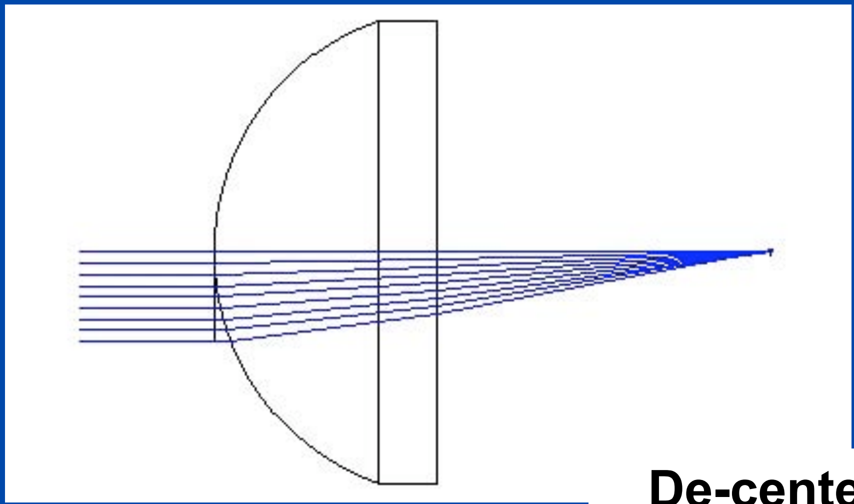
- Coma and astigmatism can be thought of as the aberrations from a de-centered bundle of spherically aberrated rays
- Ray bundle on axis shows spherical aberration only
- Ray bundle slightly de-centered shows coma
- Ray bundle more de-centered shows astigmatism
- All generated from subsets of a larger centered bundle of spherically aberrated rays
  - (diagrams follow)

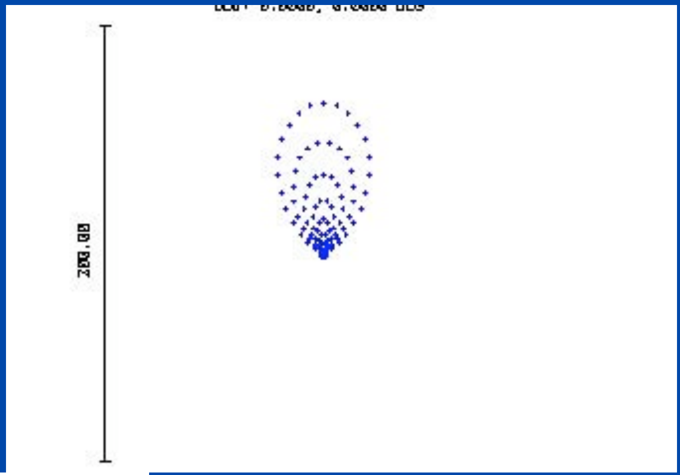
Spherical aberration as the parent of coma





Big bundle of spherically aberrated rays

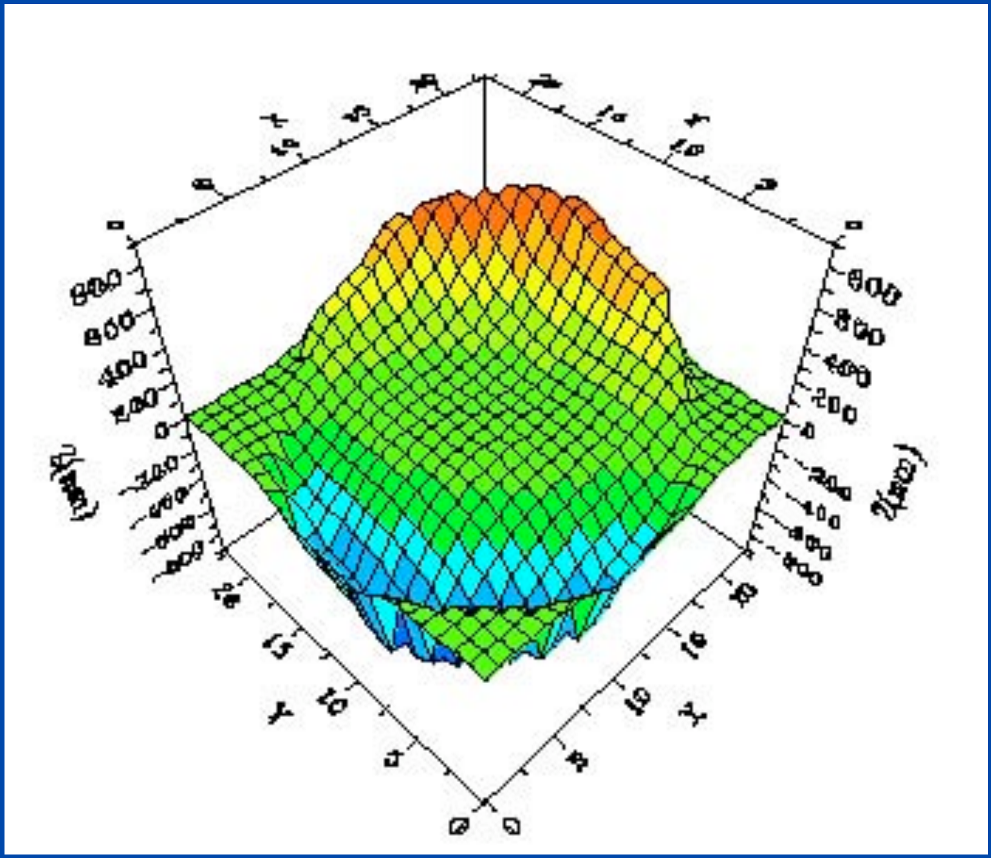




De-centered subset of rays produces coma

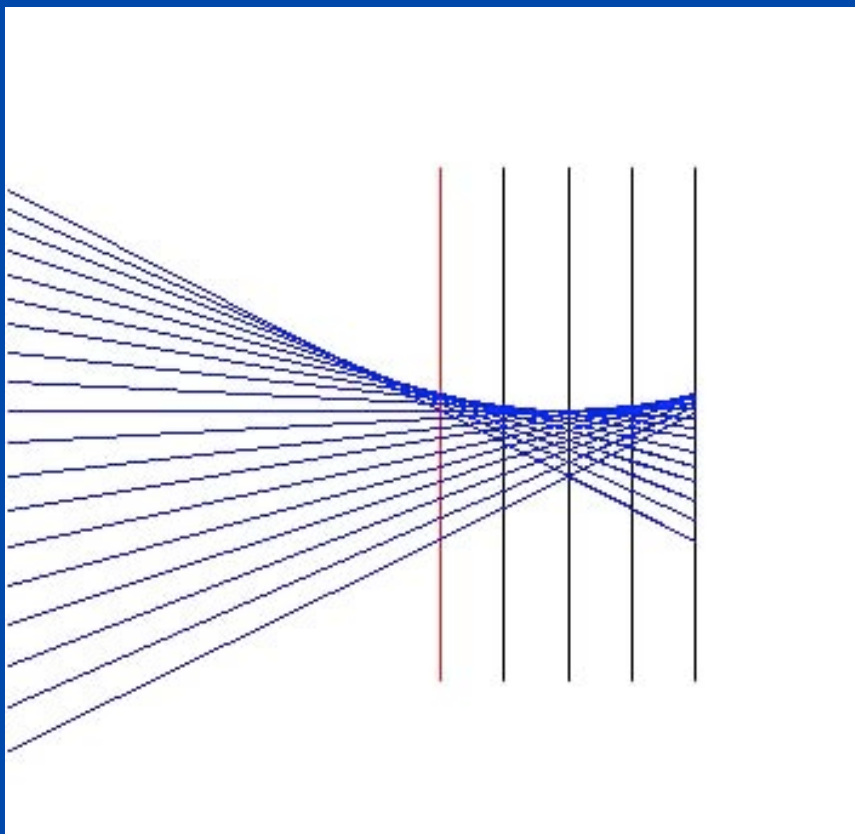
Coma

- "Comet"-shaped spot
- Chief ray is at apex of coma pattern
- Centroid is shifted from chief ray!
- Centroid shifts with change in focus!

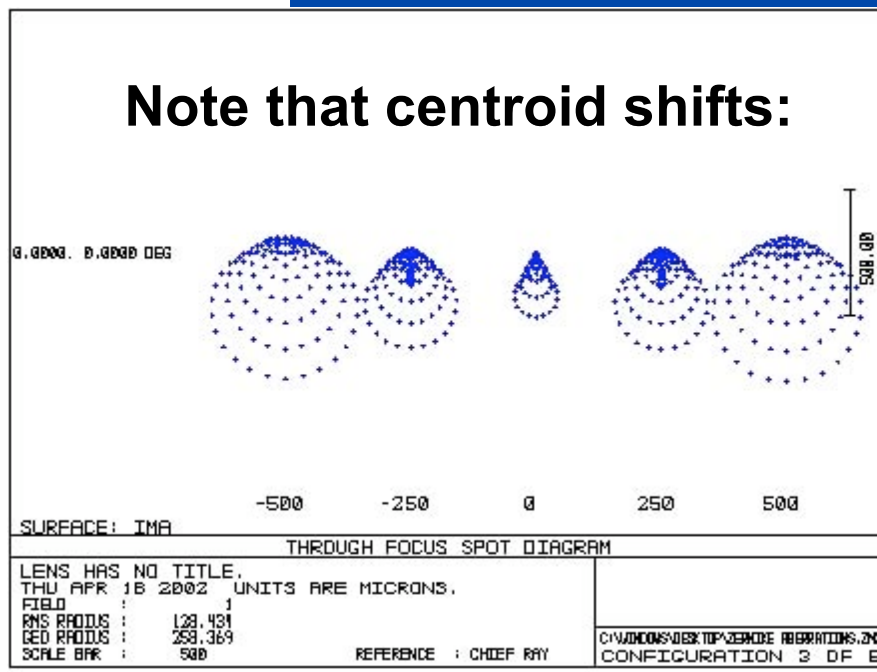


Wavefront

Coma

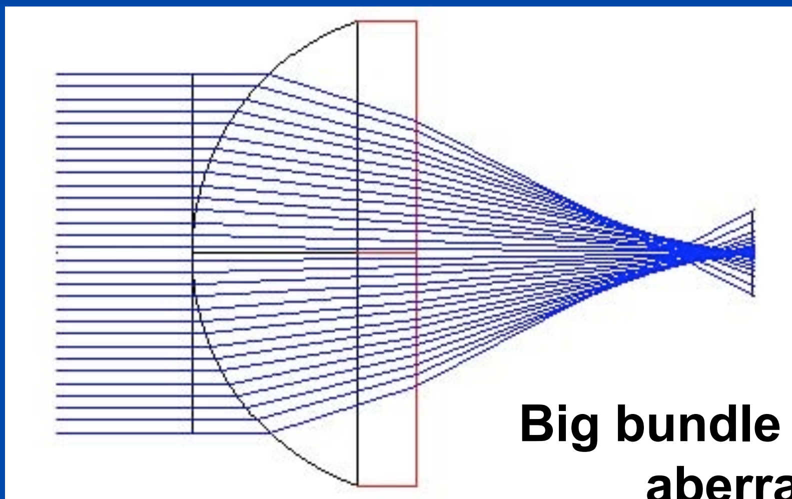


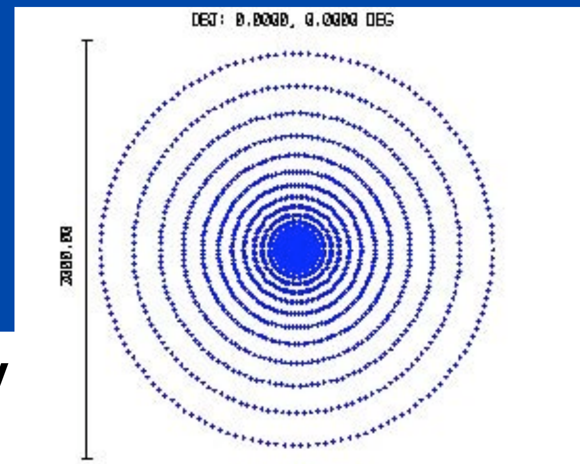
Rays from a comatic wavefront



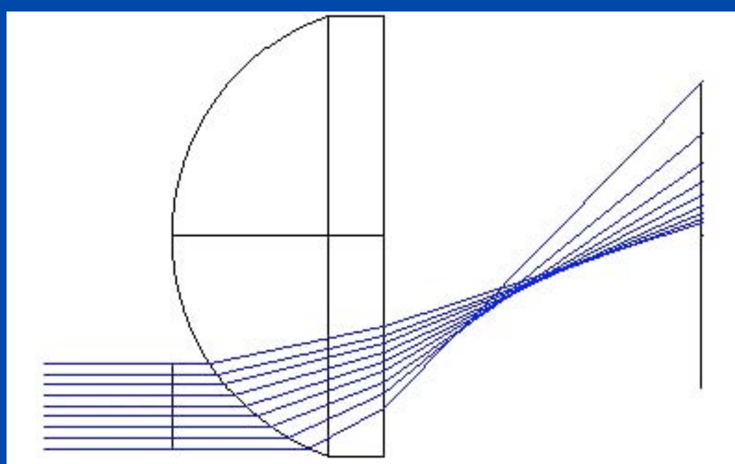
Through-focus spot diagram for coma

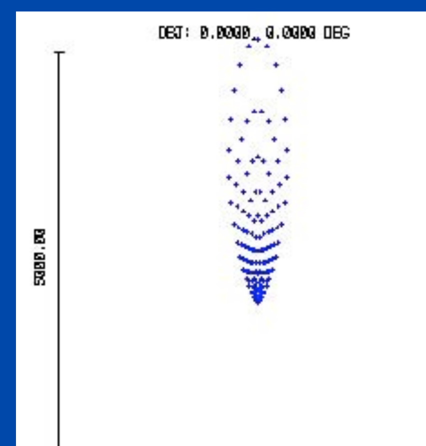
Spherical aberration as the parent of astigmatism





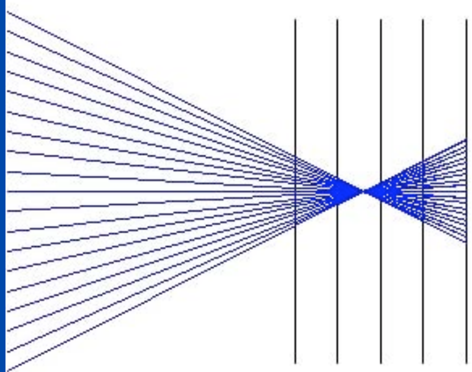
Big bundle of spherically aberrated rays



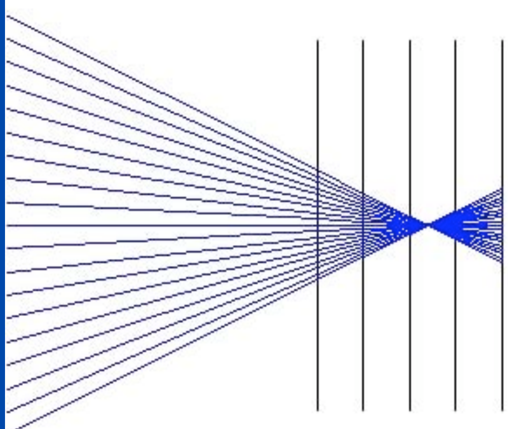


More-decentered subset of rays produces astigmatism

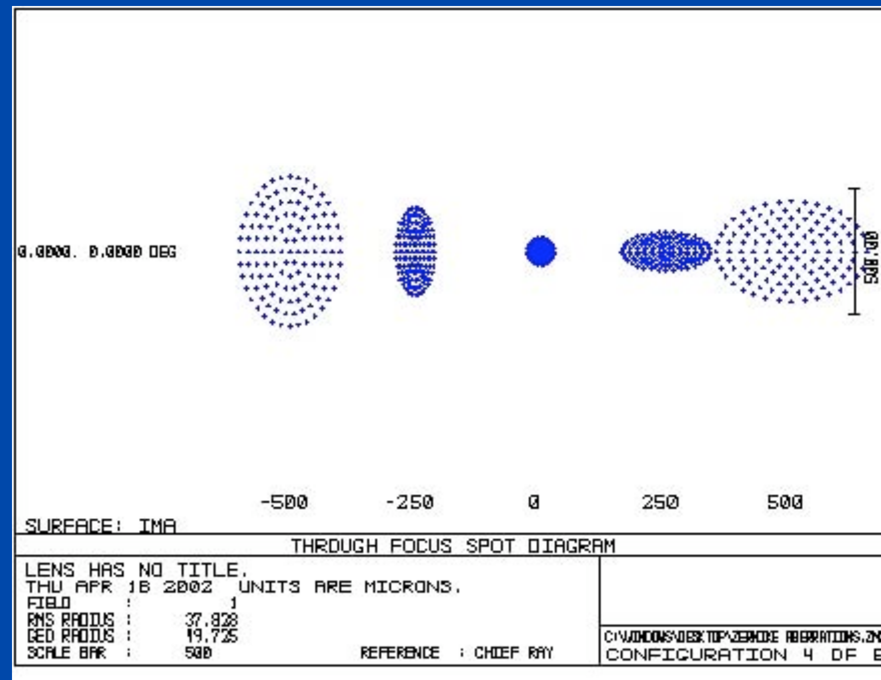
Astigmatism



Top view of rays

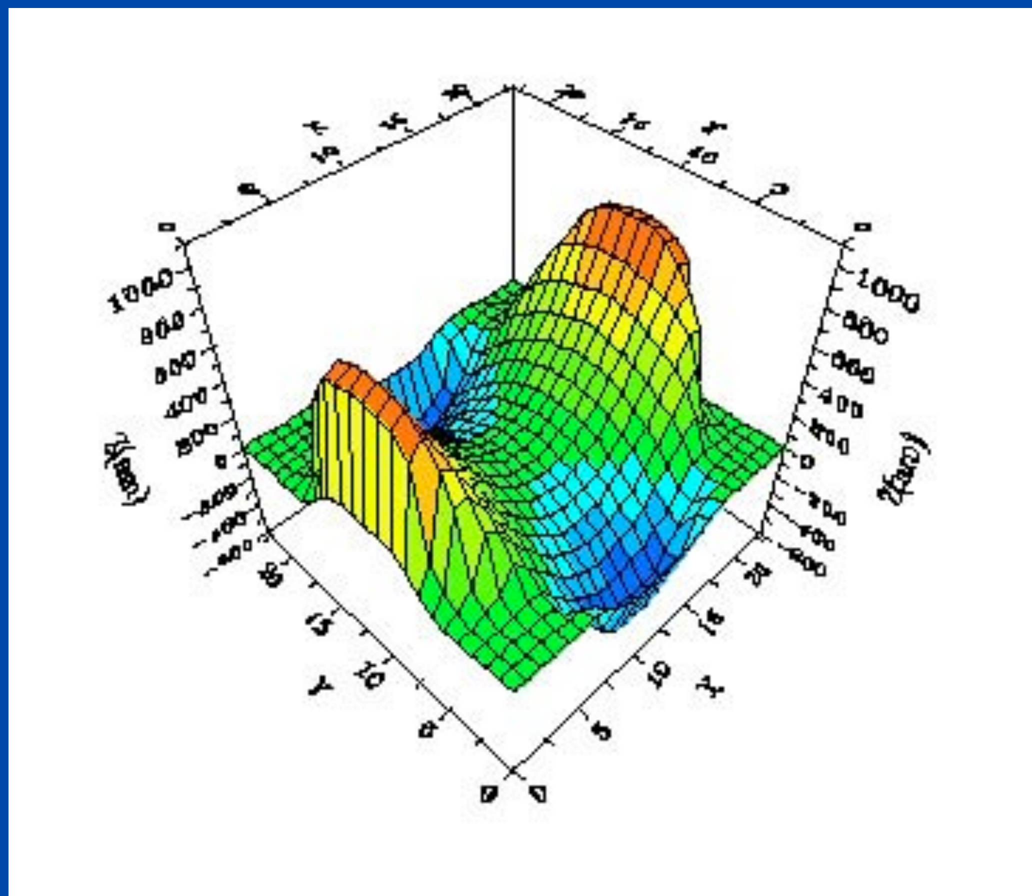


Side view of rays

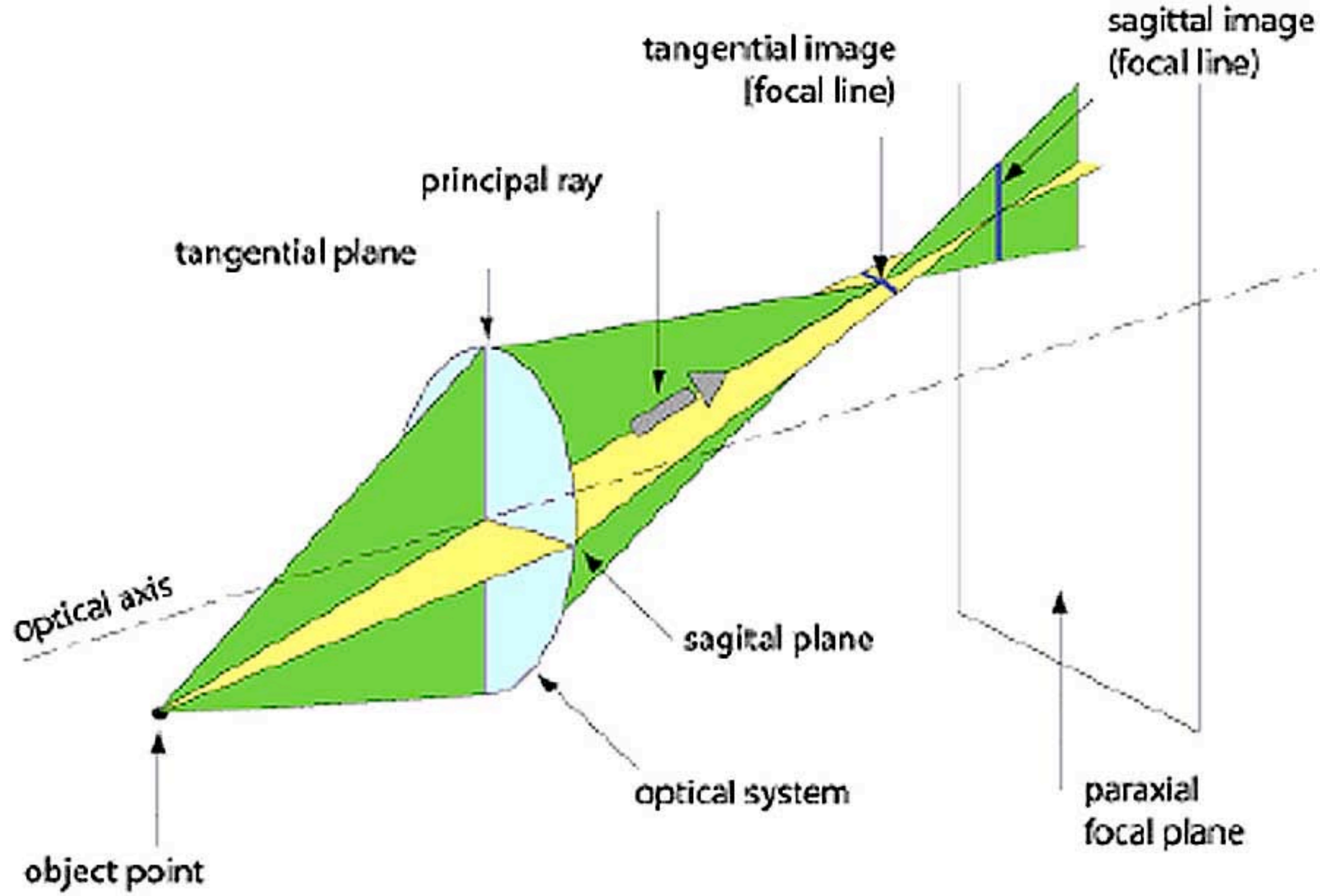


Through-focus spot diagram for astigmatism

Wavefront for astigmatism



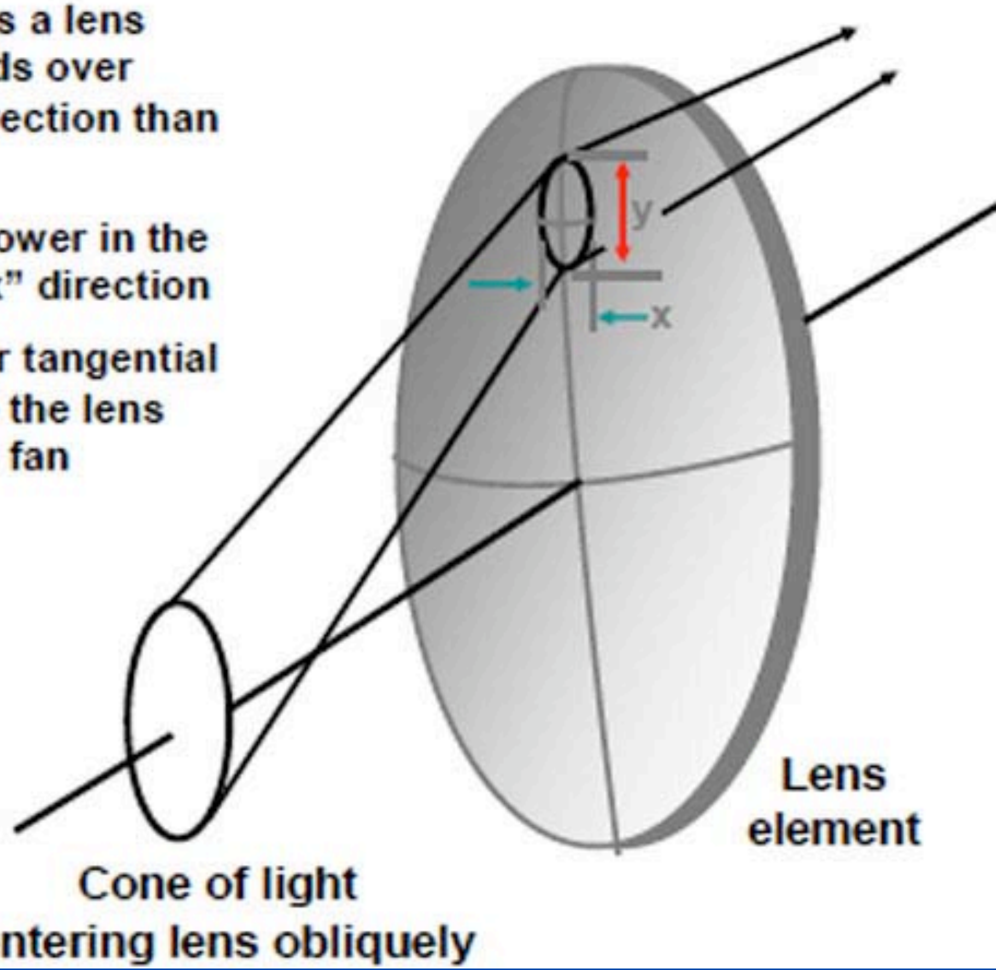
Different view of astigmatism



Astigmatism represented by sectional views

Where does astigmatism come from?

- When a cone of light enters a lens surface obliquely, it extends over more surface in the "y" direction than I the "x" direction
- This will introduce more power in the "y" direction than in the "x" direction
- The result is that the "y" or tangential ray fan will focus closer to the lens than the "x" or sagittal ray fan
- This is astigmatism



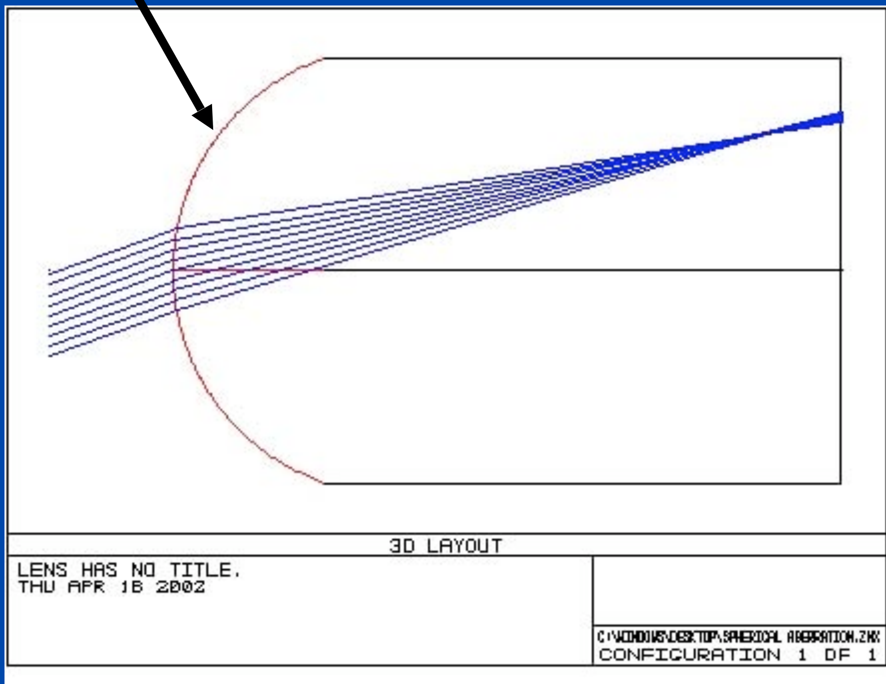
From Ian McLean, UCLA

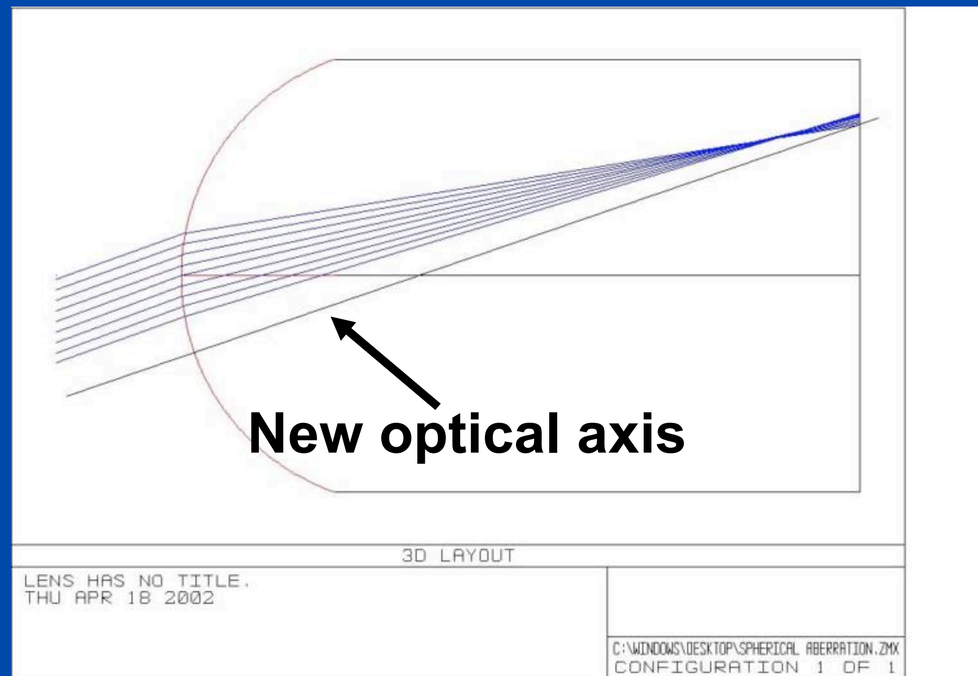
Concept Question

- How do you suppose eyeglasses correct for astigmatism?

Off-axis object is equivalent to having a de-centered ray bundle

Spherical surface





Ray bundle from an off-axis object. How to view this as a de-centered ray bundle?

For any field angle there will be an optical axis, which is ⊥ to the surface of the optic and || to the incoming ray bundle. The bundle is de-centered wrt this axis.

Zernike Polynomials

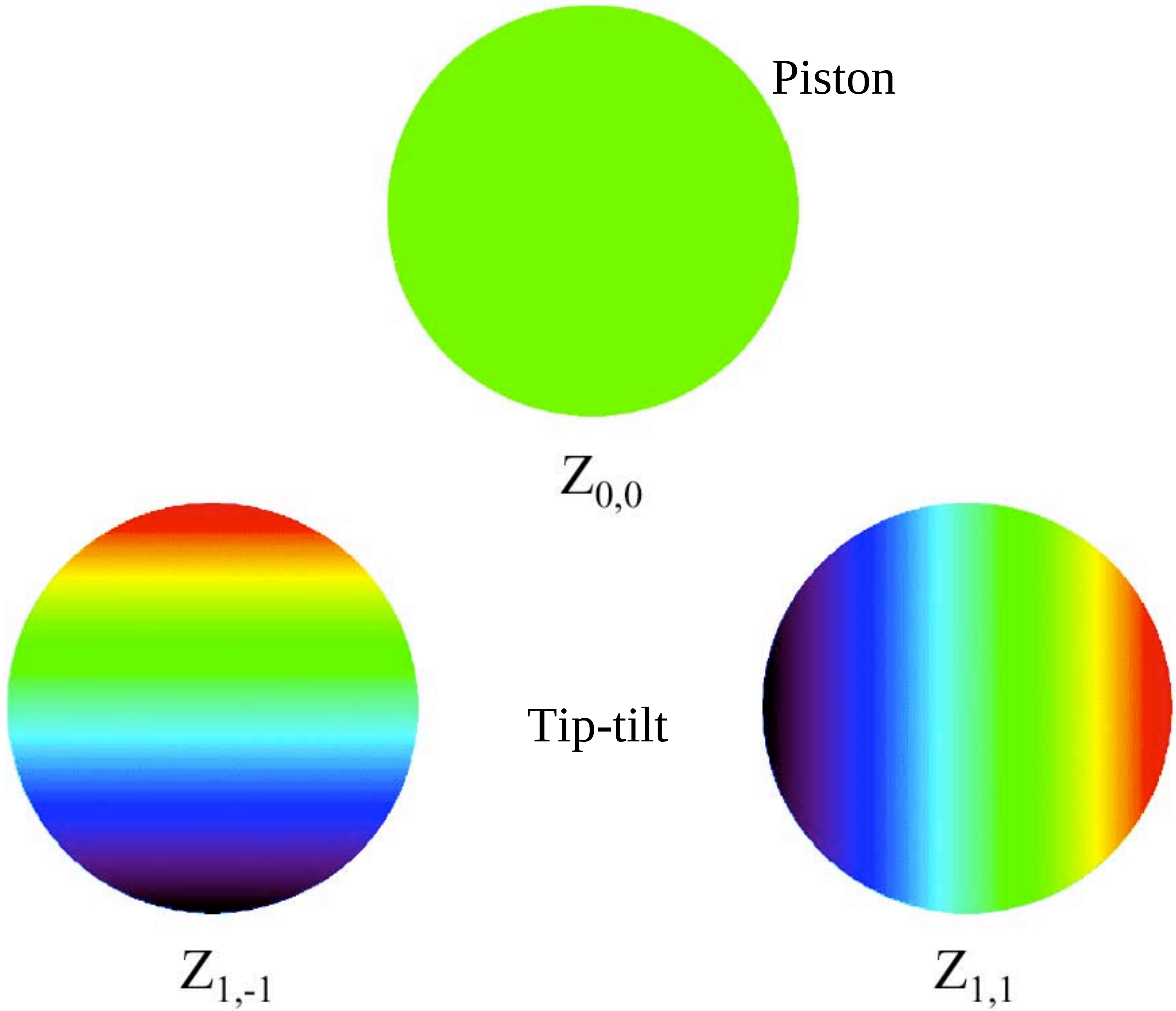
- Convenient basis set for expressing wavefront aberrations over a *circular pupil*
- Zernike polynomials are orthogonal to each other
- A few different ways to normalize – always check definitions!

Expansion of the Phase in Zernike Polynomials

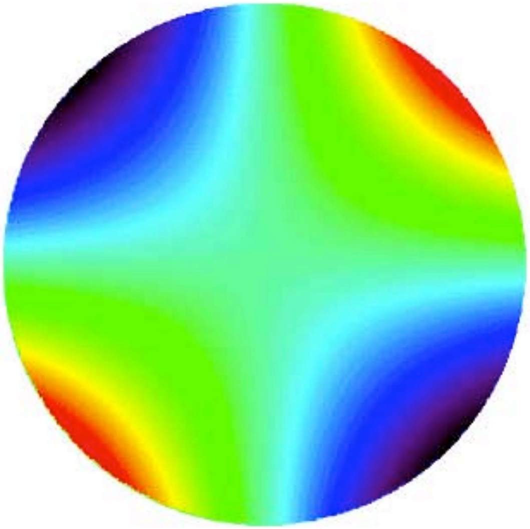
An alternative characterization of the phase comes from expanding φ in terms of a complete set of functions and then characterizing the coefficients of the expansion:

$$\varphi(r,\theta) = \Sigma\, a_{m,n} Z_{m,n}(r,\theta)$$

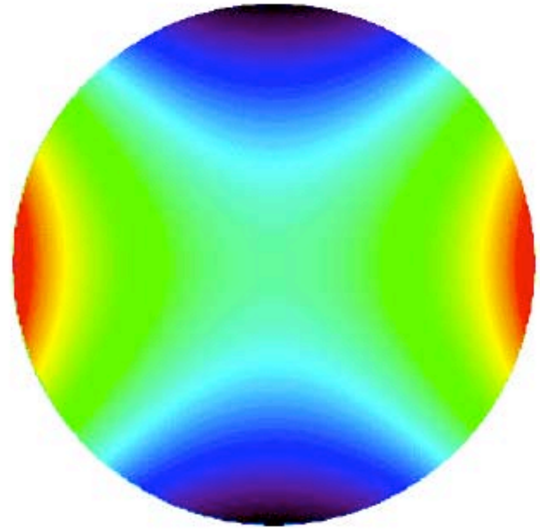
$Z_{0,0} = 1$	piston
$Z_{1,-1} = 2\, r \sin\theta$	tip/tilt
$Z_{1,1} = 2\, r \cos\theta$	tip/tilt
$Z_{2,-2} = \sqrt{6}\, r^2 \sin 2\theta$	astigmatism
$Z_{2,0} = \sqrt{3}\, (2r^2 - 1)$	focus
$Z_{2,2} = \sqrt{6}\, r^2 \cos 2\theta$	astigmatism


Piston
Z0,0
Tip-tilt
Z1,-1
Z1,1

Astigmatism (3rd order)

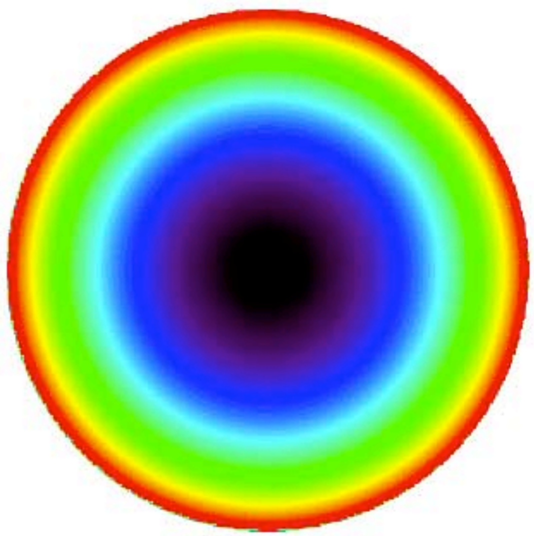


$Z_{2,-2}$

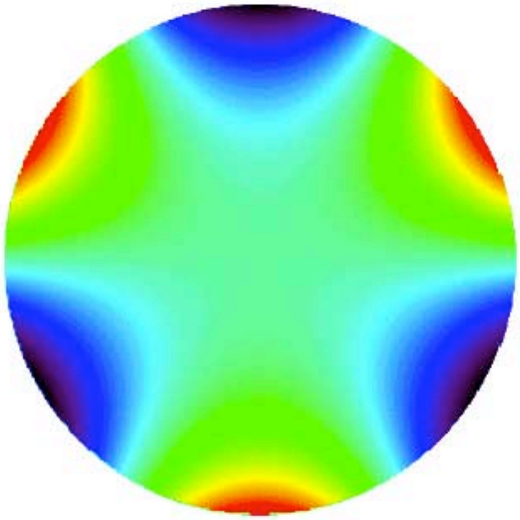


$Z_{2,2}$

Defocus

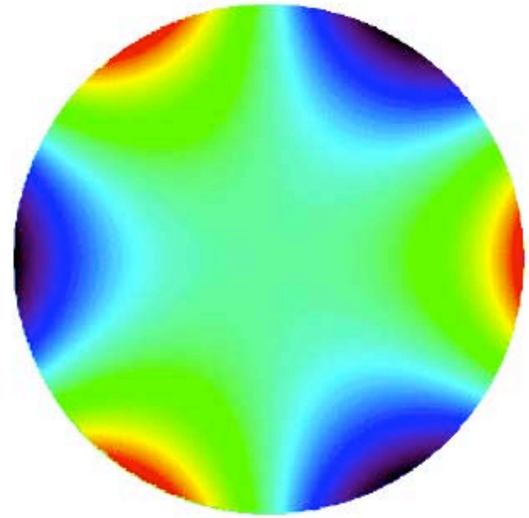


$Z_{2,0}$

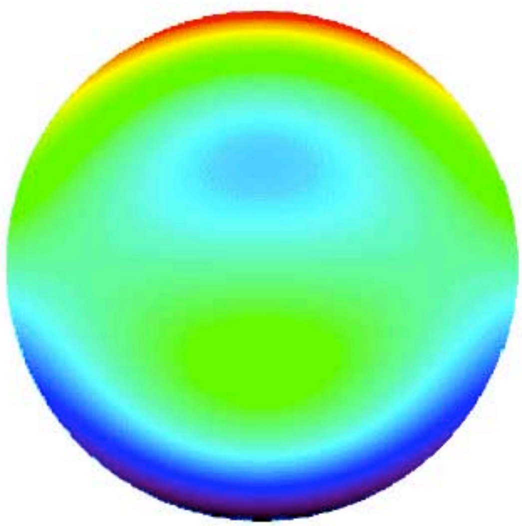


$Z_{3,-3}$

Trefoil

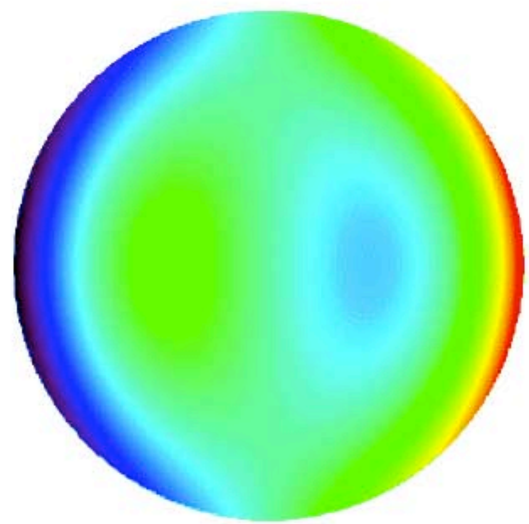


$Z_{3,3}$

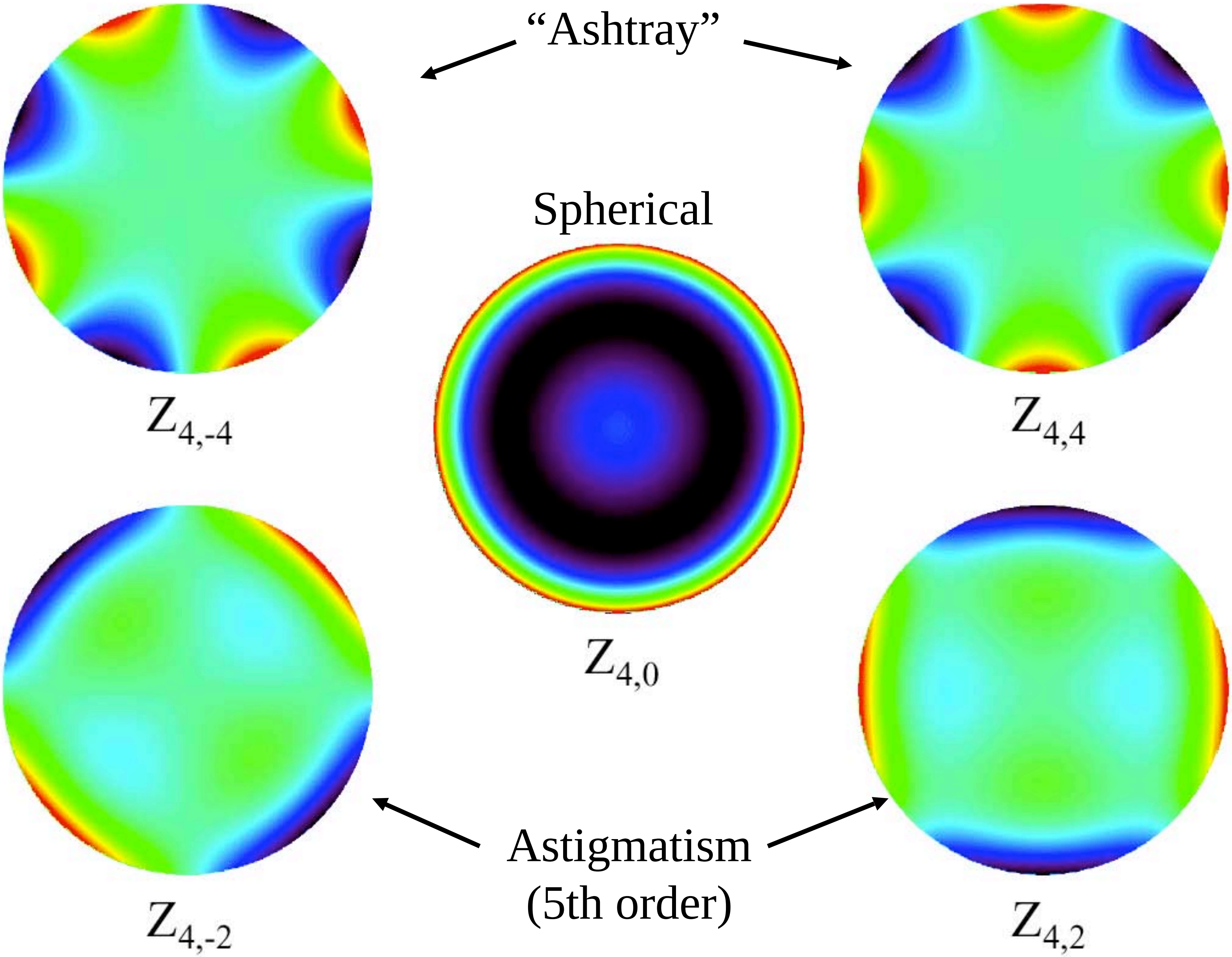


$Z_{3,-1}$

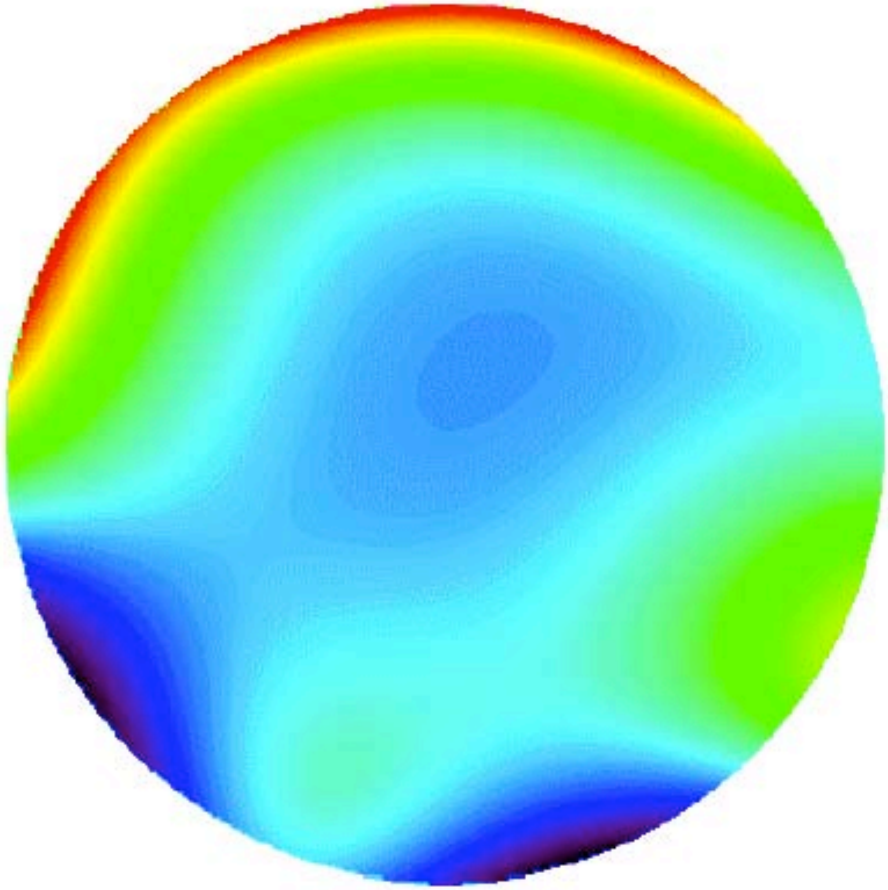
Coma



$Z_{3,1}$

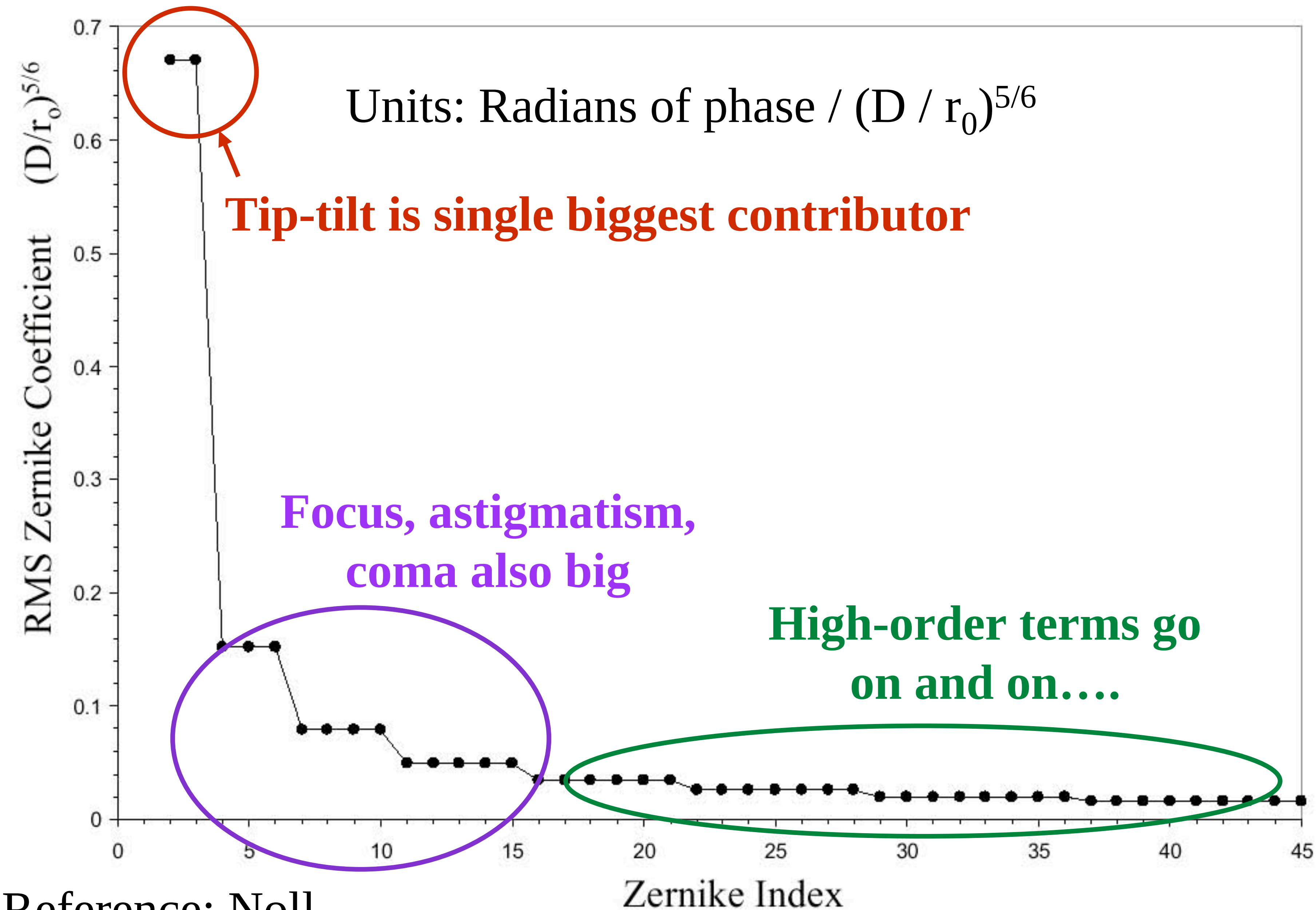


"Ashtray"
Spherical
$Z_{4,-4}$
$Z_{4,4}$
$Z_{4,0}$
$Z_{4,-2}$
Astigmatism
(5th order)
$Z_{4,2}$



Random Zernikes

Atmospheric Zernike Coefficients



Reference: Noll

Seidel polynomials vs. Zernike polynomials

- Seidel polynomials also describe aberrations
- At first glance, Seidel and Zernike aberrations look very similar
- Zernike aberrations are an orthogonal set of functions used to decompose a given wavefront ***at a given field point*** into its components
  - Zernike modes add to the Seidel aberrations the correct amount of low-order modes to minimize rms wavefront error
- Seidel aberrations are used in optical design to predict the aberrations in a design and how they will vary ***over the system's field of view***
- The Seidel aberrations have an analytic field-dependence—proportional to some power of field angle

References for Zernike Polynomials

- Books:
  - Hardy pages 95-96 READ THIS

- Pivotal Paper: Noll, R. J. 1976, "Zernike polynomials and atmospheric turbulence", JOSA 66, page 207

Review of important points

- Both lenses and mirrors can focus and collimate light
- Equations for system focal lengths, magnifications are quite similar for lenses and for mirrors
  - But be careful of sign conventions
- Telescopes are combinations of two or more optical elements
  - Main function: to gather lots of light
  - Secondary function: magnification
- Aberrations occur both due to your local instrument's optics and to the atmosphere
  - Can describe both with Zernike polynomials

Next time: Physical optics (diffraction), atmospheric turbulence

- Reading for Thursday:
  - Review geometrical optics: (this lecture, Tipler, Melles-Griot)
  - Read section on diffraction in Reader (Harvey)
- Homework: Problem set on optics will be due next Tuesday