Name:______________________________ **Date:**____________ **Per:** _______

Determining the Sizes of Stars Using the H-R Diagram Exercise

In this exercise, you are going to use an Hertzsprung-Russell Diagram to determine the luminosity and temperature of stars. Then you are going to use a relationship called Stephan's Law to calculate the stars' radii. By comparing the radii of different stars you will then find out the reason behind the names of the four categories of stars: white dwarfs, main-sequence stars, giants and supergiants.

A. Stars

Stars are born with a wide variety of ***mass***. The most massive stars are 100 times more massive than the Sun while the least massive ones are only 0.08 times the mass of the Sun. Most stars spend about 90% of their lifetimes shining due to nuclear fusion that goes on in their cores, but after awhile they evolve and begin to die. How long they live and what they evolve to become when they die depends on their mass. In fact, the mass of a star also determines its most important properties: its luminosity, temperature and radius.

A star's **luminosity**, which is how much energy is emitted per second from the star, is measured in Watts or in solar luminosities ($L_\odot$) where $1L_\odot = 3.85 \times 10^{26}$ Watt. We determine a star's luminosity by measuring its distance and its apparent brightness, which we call its apparent magnitude. Knowing those two, we can calculate its absolute magnitude, which is how bright the star would be if it were 10 parsecs away from us, and its luminosity relative to the Sun.

A star's **temperature** is the temperature of the gas on the surface of the star. We measure temperature on the Kelvin scale, in which 0 K means that an object has absolutely zero energy. Note that the temperature of the surface of a star is much lower than the temperature in the interior of the star where nuclear reactions happen. For example, the Sun's surface temperature is approximately 6,000 K, but the temperature at the center of the Sun is 15,000,000 K! That is why nuclear reactions only happen in the interiors of stars.

A star's ***radius*** is simply half the star's diameter. Stars are simply large balls of gas held together by gravity, and they are approximately spherical in shape. Radii of stars can be measured in meters, but because stars are so very large that its much more convenient to measure stellar radii in units of the Sun's radius, where $1\ R_\odot = 6.96 \times 10^8$ m.

B. The Hertzsprung-Russell Diagram

A graph of the temperature and luminosity of stars is called the Hertzsprung-Russell Diagram (H-R Diagram), which is named after two famous astronomers. Einar Hertzsprung was a Danish astronomer, and Henry Norris Russell was an American astronomer, and they both did ground-breaking work in measuring and understanding the properties of stars in the early 1900s.

C. Calculating a Star's Radius Using Stefan's Law

Stefan's Law says that for any radiating object its luminosity, temperature and radius are related by this simple formula:

$$\frac{L}{4\pi R^2} = \sigma T^4 \qquad \text{EQ \#1}$$

where L is the luminosity, R is the radius, T is the surface temperature, $\pi = 3.141$ and $\sigma = 5.671 \times 10^{-8}$ Watt/m^2 K^4. This means that if we measure the luminosity and temperature of a star then we can calculate its radius. Taking the above equation and solving for R gives us

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}} \qquad \text{EQ \#2}$$

which is the equation we can use to calculate the radius in meters. Once we have that we can convert it to solar radii by using

$$\text{R in } R_\odot = \text{R in meters} \times 1\ R_\odot / 6.96 \times 10^8 \text{ m.} \qquad \text{EQ \#3}$$

D. An Example: The Sun

Lets do an example and calculate the Sun's radius. Looking at the HR Diagram, we see that the Sun's luminosity is 1 $L_\odot = 3.85 \times 10^{26}$ Watt and its temperature is approximately 6,000 K. To calculate its radius, we use EQ #2:

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

$$R = \sqrt{\frac{3.85\times10^{26}\ \text{Watt}}{4\times3.141\times5.671\times10^{-8}\ \text{Watt/m}^2\text{K}^4\times(6000\ \text{K})^4}}$$

$$R = \sqrt{4.17\times10^{17}\ \text{m}^2}$$

$$R = 6.46\times10^{8}\ \text{m}$$

Now convert the radius from units of m to units of solar radius:

$$\text{R in } R_\odot = \text{R in meters} \times 1\ R_\odot / 6.96 \times 10^8 \text{ m}$$

$$R = 6.46 \times 10^8 \text{ m} \times 1\ R_\odot / 6.96 \times 10^8 \text{ m}$$

$$R = 0.93\ R_\odot \approx 1\ R_\odot.$$

E. Calculate the Radius for Your Star

Star Name: ___________________

Star's Category (Circle One): White Dwarf Main Sequence Giant Supergiant

Using the HR Diagram, record below your star's luminosity in solar units and its temperature in degrees K, but remember that the temperature axis is non-linear and backward so that hotter objects are to the left and cooler objects are to the right.

Luminosity, L = _____________ $L_\odot$

Temperature, T = _____________ K

Convert your star's luminosity from units of solar luminosities to Watts:

Luminosity, L = ___________ $L_\odot$ x 3.85 x 10^{26} Watt / 1$L_\odot$

L = ___________ Watt

Now calculate the star's radius using EQ #2:

$$R = \sqrt{\frac{L}{4\pi\sigma T^4}}$$

$$R = \sqrt{\frac{\rule{6cm}{0.4pt}\ \text{Watt}}{4 \times 3.141 \times 5.671 \times 10^{-8}\ \text{Watt/m}^2\text{K}^4 \times (\rule{3cm}{0.4pt}\ \text{K})^4}}$$

$$R = \sqrt{\rule{4.5cm}{0.4pt}\ \text{m}^2}$$

$$R = \rule{4.5cm}{0.4pt}\ \text{m}$$

Now convert the radius from units of meters to units of solar radius using EQ #3,

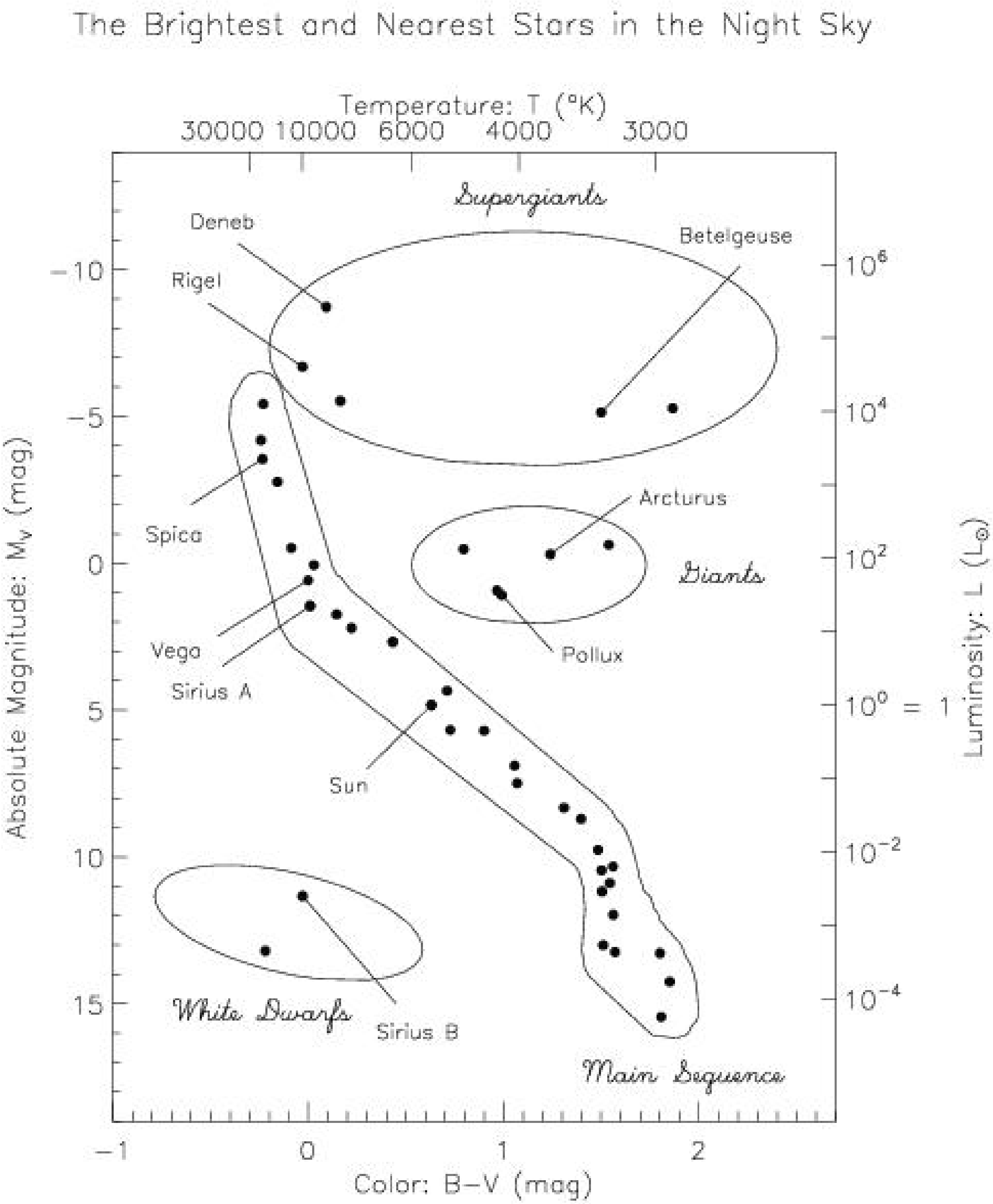
$$R \text{ in } R_\odot = R \text{ in meters} \times 1\, R_\odot / 6.96 \times 10^8 \text{ m}$$

R = ____________________ m x 1 $R_\odot$ / 6.96 x 10^8 m

R = ________________ $R_\odot$

F. Compare Your Results

Once the students in your group have finished calculating the radius, compare your answers, resolve any differences and inform the teacher of your results. Your teacher will collect the results and then you will analyze them to understand how the radii of stars vary among the different categories of stars.


The Brightest and Nearest Stars in the Night Sky
Temperature: T (°K)
30000 10000 6000 4000 3000
Supergiants
Deneb
Rigel
Betelgeuse
Spica
Arcturus
Giants
Pollux
Vega
Sirius A
Sun
White Dwarfs
Sirius B
Main Sequence
Absolute Magnitude: M_V (mag)
−10
−5
0
5
10
15
Luminosity: L ($L_\odot$)
10^6
10^4
10^2
10^0 = 1
10^{-2}
10^{-4}
−1 0 1 2
Color: B−V (mag)

Data & Results

You can write a blank version of the table (see below) on the blackboard and have a representative from each group come up and fill in the entries for their star. Then have the class examine the trends in the radii. The answers are on the following page.

Star Name	Category	L ($L_\odot$)	T (K)	R (m)	R ($R_\odot$)
Betelgeuse					
Deneb					
Rigel					
Arcturus					
Pollux					
Spica					
Vega					
Sirius A					
Sun					
Sirius B					