SYMMETRIES, REDUCTIONS AND RELATIVE EQUILIBRIA FOR A GYROSTAT IN THE THREE-BODY PROBLEM

S. Ferrer[1], F. Mondéjar and A. Vigueras[2]

[1]*Departamento de Matemática Aplicada, Universidad de Murcia,*
30071 Espinardo (Murcia), Spain. E-mail: sferrer@fcu.um.es
[2]*Departamento de Matemática Aplicada y Estadística, Universidad Politécnica de Cartagena,*
30203 Cartagena (Murcia), Spain. E-mail: Antonio.Vigueras@upct.es

In memoriam of our colleague F. Mondéjar

Some papers within differential geometry frame show a new interest in the study of configurations of relative equilibria in different problems of roto-translational motion of celestial bodies. In the problem of three rigid bodies Vidiakin (1977) and Duboshine (1984) proved the existence of Euler and Lagrange configurations of equilibria when the bodies possess symmetries (for a recent review see Zhuravlev and Petrutskii (1990). In order to study the configurations of equilibria of the general problem of three rigid bodies, from a global geometrical point of view, it is natural to consider first the problem when two bodies have spherical distribution of mass; Fanny and Badoui (1998) study this problem in terms of the global variables in the unreduced problem. It is clear, as the papers of Maciejewski (1995), Mondéjar and Vigueras (1999) show, that to work in the reduced system if the problem has symmetries, produces natural simplifications in the conditions of the equilibria, and then more general results can be obtained. This is the approach we will follow to study the problem of three bodies when two are spherical and the other is a gyrostat. Using the symmetries of the translational and rotational group possessed by the system, me perform a reduction process in two steps, giving explicitly at each step the Poisson structure of the reduce system. We make global considerations about the relative equilibria of the problem and give a general classification of them. Finally, me restrict to the zero order approximation dynamics and a complete study of the relative equilibria is made. We enlarge some results obtained in Fanny and Badoui (1998). We note also that the reduction procedure presented here applies immediately to rigid body case when me take the gyrostatic momentum be zero.