Schwarzschield's nonequatorial periodic motion about an asteroid modeled as a triaxial rotating ellipsoid

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For the first time existence of three-dimensional periodic motion of Schwarzschiel's type in the vicinity of a rotating triaxial ellipsoid was proved by Yu.V.Batrakov in 1957. A homogeneous uniformly rotating ellipsoid of near-spherical shape was considered there. Truncated harmonic expansion of its gravity field included up to the second harmonic. The current study differs from that investigation mainly by using a closed-form expression of a triaxial ellipsoid's gravity field

$$V = k^2 \pi \rho abc \int\limits_{\lambda}^{+\infty} \left(1 - \frac{x^2}{a^2+s} - \frac{y^2}{b^2+s} - \frac{z^2}{c^2+s} \right) \frac{ds}{R(s)} \,,$$

where $R(s) = \sqrt{(a^2+s)(b^2+s)(c^2+s)}$.

This allows researching ellipsoids of rather elongated shape. The particle motion equations in rotating body-fixed frame are expressed through the Delaunay's canonical elements. The searched periodical non-planar solutions are of Schwarzschiel's type as their period is not necessarily equal to that of an inducing unperturbed motion. The numerical technique to search for such the solutions is developed based on the Poincare's theory of periodic solutions existence conditions. The computed solutions have been improved using numerical integrating by Everhardt's method. Some tables demonstrate found symmetrical and asymmetrical solutions for some commensurabilities and ellipsoid's shapes with semi-axies 70,60,50 km; 140,60,25 km, etc. Several figures demonstrate (for certain initial values of the Delaunay's elements and ellipsoid's models and commensurabilities) how inclination of possible initial orbit which induces the periodic motion depends on its eccentricity. In particularly, it may be noticed from these figures that both the ellipsoid's shape and particle's distance from the ellipsoid (which depends on the chosen commensurability) essentially affect the initial inclination of the periodic orbit. It is pointed to some regularities found for mutual dependence of initial elements of periodic motion. Paper includes comprehensive literature on this subject.