Lyapunov Stability for Lagrange equilibria of Orbiting Dust

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We examine the basic dynamics associated with a simple model of a dust particle interacting with a planetary gravitational field and solar radiation pressure. No attempt is made to model a particular planetary ring because our goal is to uncover generic properties of the model. Thus a number of important details are omitted, including; plasma fluctuations, particle-particle collisions, time dependent variations in the magnetosphere, etc. This model matches with that of a Rydberg atom in combined magnetic and circularly polarized microwave fields and we find the presence of global equilibrium points that are analogous to the Lagrangian equilibria of the circular restricted three-body problem. The dynamics around these points proves to be of special interest, so that the stability of them is an important question.

We perform linear stability analysis of the equilibrium points in order to establish stabilty conditions in terms of the two free parameters of the problem. However, linear stability does not ensures Lyapunov stability and further analysis is needed. To go further in the analysis, we apply a theorem due to Arnold that ensures Lyapunov stabilty almost for every pair of the free parameters but for some resonant cases. These resonant cases are analyzed by studying the phase flow on the reduced phase space after normalization. We use extended Lissajous variables and Morse theory.