Multi-dimensional Fourier Transformation of Planetary Disturbing Function

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It is well known that Fourier expansions of planetary disturbing function can be computed numerically with the help of numerical Fourier analysis (see, e.g., Brouwer, D., Clemence, G.M.: 1961 'Methods of Celestial Mechanics', Academic Press, N.Y.). What is making this approach advantageous nowadays is fast computers and Fast Fourier Transformation (FFT) algorithm. The FFT algorithm has been introduced into modern practice by Cooley and Tukey in 1965. Retrospectively it has become known that the FFT algorithm was invented independently by a dozen of individuals starting from Gauss in 1805 (Heideman, M.T., Johnson, D.N., Burrus, C.S.: 1985 'Gauss and the History of the Fast Fourier Transform'. Archive for History of Exact Sciences, **34**, 265–277). It was exciting also to learn that Gauss has invented the algorithm to compute Fourier expansions of the planetary disturbing function. Most general 5-dimensional Fourier expansion of the planetary disturbing function in standard notations reads:

$$R=\frac{G\,\mathcal{M}'}{a'}\sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\sum_{m=0}^{\infty}B_{ijklm}(\alpha,e,e',i,i')\,\cos(i\,M+j\,M'+k\,\omega+l\,\omega'+m(\Omega-\Omega')),$$

$\alpha = a/a'$ being the ratio of the two semi-major axes. Numerical Fourier transformation of R allows one to compute coefficients B_{ijklm} for any admissible numerical values of α, e, e', i, i'. Numerical computation of these 5-dimensional series has become feasible on a typical server-class computer quite recently. For practical calculations we used SGI Origin2000 with 48 R10000 processors running at 195MHz. On that computer we used up to 8Gb RAM for a single calculation. To give an example, in order to compute series (1) for the perturbations of Jupiter on Veritas ($\alpha\approx 0.609$, $e\approx 0.1009$, $e'\approx 0.0485$, $i\approx 9^\circ 16'$, $i'\approx 1^\circ 18'$) with an accuracy of 10^{-14} the FFT size should be taken as $56\times 52\times 48\times 24\times 208$ which is required about 5.4 Gb RAM and 30 minutes of computing time in single processor mode. In this example, the number of coefficients B_{ijklm}, absolute value of which is greater than 10^{-14}, is $1\,083\,126$. Our experience shows that the numerical calculation even of the most general Fourier expansions of the planetary disturbing function is quite feasible and can become an useful tool of celestial mechanics in the nearest future. Further details and examples will be given in the presentation.