Solving a Gylden–Meščerskij System in Delaunay–Scheifele–Like Variables

Luis Floría[1]

[1] *Grupo de Mecánica Celeste I.*
Departamento de Matemática Aplicada a la Ingeniería.
E. T. S. de Ingenieros Industriales.
Universidad de Valladolid.
E – 47 011 Valladolid, Spain. E-mail: luiflo@wmatem.eis.uva.es

We deal with the *nonstationary two–body problem* in Hamiltonian formulation.

A *Gylden system* (Deprit 1983, *Celest. Mech.* **31,** 1–22) is a two–body Kepler problem in which the Keplerian coupling parameter $\mu \equiv \mu(t)$ undergoes variations in time.

In some previous papers (Floría: *Celest. Mech. and Dyn. Astron.* **68,** 1997, 75–85; *Impact of Modern Dynamics in Astronomy*, Proceedings of the IAU Colloquium No. 172, Namur, Belgium, July 1998, pp. 461–462, Kluwer 1999, Henrard & Ferraz–Mello, Eds.), inspired by the treatment of time–varying Delaunay transformations due to Deprit (1983, §3), we considered time–dependent canonical transformations of the TR–type in extended phase space (Deprit 1981, *Celest. Mech.* **23,** 299–305) that generalize the classical transformations to Delaunay–Scheifele (DS) variables (originally devised to reduce perturbed, stationary, Keplerian systems.) We applied these mappings to a generic Hamiltonian characterizing a class of perturbed Gylden–like systems reduced to the orbital plane.

As a further development of our previous analytical considerations, we study the solution of a nonstationary two–body problem, with a time–varying gravitational parameter $\mu(t)$. We restrict ourself to the consideration of $\mu(t)$ subjected to variations according to the Meščerskij's laws, in which case the problem is known to be integrable.

The mathematical law of change of μ known as the *first Meščerskij law* leads to the integrability case of Gylden systems most often considered in Celestial Mechanics. This integrable and integrated model, with exact analytical solution, has been thoroughly analysed, and can be used as a first approximation in the investigation of more complicated perturbed Gylden systems.

Due to the significance of the Gylden–Meščerskij problem in astronomical, astrophysical and cosmological studies (e.g., isotropic mass loss in binary or multiple star systems), and as a preliminary step prior to the treatment of problems of perturbed motion involving time–dependent gravitational–type systems, described as *perturbed Gylden problems*, we find it interesting and appropriate to complete the picture by developing the corresponding study and solution in terms of canonical variables within a generalized DS–type formulation.

Key Words: nonstationary two–body problem, time–varying Keplerian coupling parameter, Gylden systems, Meščerskij's laws, Delaunay–Scheifele–like canonical variables, uniform treatment of two–body motion.