Progress of the perturbation theory of the attitude dynamics

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The problem of constructing the analytical theory of the rotational motion of the Earth had one from central place in celestial mechanics. Starting from classical papers on this subject the equations of rotational motion in the osculating elements similar to Andoyer and angle-action variables have been used (Laplace, 1825; Pontecoulant, Tisserand, 1891; Andoyer, 1923; et. al.). But by construction of the force function of the problem in these variables authors as usually used the approximate developments using additional assumption about small eccentricities of the ellipsoid of inertia. Analytical solution of the problem had restricted character. In Kinoshita paper (1977) by construction of the Earth rotation theory similar simplifications were used (only constant of precession was presented in explicit form in term of elliptic integrals). Full formulae for first-order perturbations of the rotational motion of the satellite (Earth) in the gravitational field of the perturbing body (the Moon and the Sun) in angle-action variables (for Euler and Euler-Chandler unperturbed motions) were constructed (Barkin, 1993, 1998; Ferrandiz, Barkin and Getino, 1995). Amplitudes of the all perturbations of the first order were presented in terms of the elliptic functions and integrals of the action variables. It means that the analytical theory is applicable for study of the attitude motion of the natural and artificial celestial bodies with arbitrary dynamical structure. We have adopted similar theory for study of the rotation of the deformable celestial bodies. The new form of the canonical and noncanonical equations in Andoyer and angle-action variables were suggested. These variables were introduced on the basis of the integrable Euler-Chandler problem about attitude motion elastic body deformed by its own rotation (Ferrandiz, Barkin and Getino, 1995). Unperturbed motion takes into account the elastic property and reduced to the classical Euler-Poinsot problem but with special changed principal moment of inertia. The properties of the Earth Euler-Chandler unperturbed motion were described in details (Chandler period, eccentricity of the pole trajectory, non-uniform pole motion and others). The problem about rotation of the isolated deformable body with changeable in the time shell was studied. The components of the shell tensor of inertia are definite conditionallyperiodic functions of the time. The analytical formulae for the secular and periodic perturbations were obtained in Andoyer, angle-action and classical Euler variables. In general these formulae were obtained for arbitrary parameters of the considered unperturbed motion (for example for arbitrary unperturbed value of the angle between angular moment and polar body axis). The observed pole drift and acceleration of the Earth diurnal rotation in the last century were explained on the basis of this solution by mechanism of the relative small oscillations of the Earth lower and upper shells. New analytical formula was obtained for tidal deceleration of the Earth. Tidal perturbations of the pole motion and diurnal Earth rotation were tabulated. The analytical formulae for the precession constant and add terms to the Chandler period and diurnal rotation due to gravitational attraction of the Moon and Sun were obtained in terms of the elliptical functions and integrals. The perturbation theory of the Earth rotation was constructed in the elastic Andover variable and angle-action variables. Perturbations of the Earth rotation due to the tidal and non-tidal variations of the Earth tensor of inertia in gravitational field of the Moon and Sun have been obtained.